Άλγεβρα Α΄ Λυκείου
Εργασία Χριστουγέννων

Λύσεις

1. α) $(\alpha+4)^2+(2\alpha+3)^2=\alpha^2+(2\alpha+5)^2 \Leftrightarrow \alpha^2+8\alpha+16+4\alpha^2+12\alpha+9=\alpha^2+4\alpha^2+20\alpha+25 \Leftrightarrow$
$5\alpha^2+20\alpha+25=5\alpha^2+20\alpha+25$ ισχύει

β) $(\alpha-3\beta)^2-(\beta-3\alpha)^2+8(\alpha-\beta)(\alpha+\beta)=\alpha^2-\cancel{6\alpha\beta}+9\beta^2-\beta^2+\cancel{6\alpha\beta}-9\alpha^2+8(\alpha^2-\beta^2)=$
$=8\beta^2-8\alpha^2+8\alpha^2-8\beta^2=0$

γ) $(x+2y)^2-(2x+y)^2=x^2+\cancel{4xy}+4y^2-4x^2-\cancel{4xy}-y^2=3y^2-3x^2=3(y+x)(y-x)$

δ) $2(2x-1)^3-(x-2)(4x+1)^2=2(8x^3-12x^2+6x-1)-(x-2)(16x^2+8x+1)=$
$\cancel{16x^3}-24x^2+12x-\cancel{2}-\cancel{16x^3}-8x^2-x+32x^2+16x+\cancel{2}=27x$

2. α) $9x^2-25=(3x-5)(3x+5)$

β) $(2x+3)^2-16x^2=(2x+3-4x)(2x+3+4x)=(3-2x)(6x+3)=3(3-2x)(2x+1)$

γ) $3x^3-3x=3x(x^2-1)=3x(x-1)(x+1)$

δ) $x^4-1=(x^2-1)(x^2+1)=(x-1)(x+1)(x^2+1)$

ε) $x^3-1=(x-1)(x^2+x+1)$

στ) $x^3+8=(x+2)(x^2-2x+4)$

ζ) $5x^3-125x=5x(x^2-25)=5x(x-5)(x+5)$

η) $x^2-y^2-x+y=(x-y)(x+y)-(x-y)=(x-y)(x+y-1)$

θ) $x^2-2xy+y^2-9=(x-y)^2-3^2=(x-y-3)(x-y+3)$

3. α) $\dfrac{8x^3+1}{2x+1}=\dfrac{\cancel{(2x+1)}(4x^2-2x+1)}{\cancel{2x+1}}=4x^2-2x+1$

β) $\dfrac{x^3-x}{x^3+2x^2+x}=\dfrac{\cancel{x}(x^2-1)}{\cancel{x}(x^2-2x+1)}=\dfrac{\cancel{(x-1)}(x+1)}{(x-1)^{\cancel{2}}}=\dfrac{x+1}{x-1}$

γ) $\dfrac{1}{x+y}-\dfrac{1}{x-y}-\dfrac{2y}{y^2-x^2}=\dfrac{1}{x+y}-\dfrac{1}{x-y}+\dfrac{2y}{x^2-y^2}=\dfrac{1}{x+y}-\dfrac{1}{x-y}+\dfrac{2y}{(x-y)(x+y)}=$
$\dfrac{x-y}{(x-y)(x+y)}-\dfrac{x+y}{(x-y)(x+y)}+\dfrac{2y}{(x-y)(x+y)}=\dfrac{x-y-x-y+2y}{(x-y)(x+y)}=0$

δ) $\left(\dfrac{\alpha^3-\beta^3}{\alpha^3-\alpha\beta^2}\cdot\dfrac{\alpha-\beta}{\alpha^2+\alpha\beta+\beta^2}\right):\dfrac{1}{\alpha}=\left(\dfrac{(\alpha-\beta)(\alpha^2+\alpha\beta+\beta^2)}{\alpha(\alpha^2-\beta^2)}\cdot\dfrac{\alpha-\beta}{\alpha^2+\alpha\beta+\beta^2}\right)\cdot\alpha=$

$$\left(\frac{\cancel{(\alpha-\beta)}\cancel{(\alpha^2+\alpha\beta+\beta^2)}}{\alpha\cancel{(\alpha-\beta)}\cancel{(\alpha+\beta)}}\cdot\frac{\cancel{\alpha-\beta}}{\cancel{\alpha^2+\alpha\beta+\beta^2}}\right)\cdot\alpha=\frac{1}{\alpha}\cdot\alpha=1$$

ε) $\dfrac{\alpha^{-3}-\alpha^{-2}+\alpha^{-1}}{\alpha^{-4}-\alpha^{-3}+\alpha^{-2}}=\dfrac{\frac{1}{\alpha^3}-\frac{1}{\alpha^2}+\frac{1}{\alpha}}{\frac{1}{\alpha^4}-\frac{1}{\alpha^3}+\frac{1}{\alpha^2}}=\dfrac{\frac{1}{\alpha^3}-\frac{\alpha}{\alpha^3}+\frac{\alpha^2}{\alpha^3}}{\frac{1}{\alpha^4}-\frac{\alpha}{\alpha^4}+\frac{\alpha^2}{\alpha^4}}=\dfrac{\frac{1-\alpha+\alpha^2}{\alpha^3}}{\frac{1-\alpha+\alpha^2}{\alpha^4}}=\dfrac{\alpha^4\cancel{(1-\alpha+\alpha^2)}}{\alpha^3\cancel{(1-\alpha+\alpha^2)}}=\alpha$

4. $\left(x+\dfrac{1}{x}\right)^2=x^2+2x\cdot\dfrac{1}{x}+\dfrac{1}{x^2}=x^2+\dfrac{1}{x^2}+2=7+2=9\Leftrightarrow x+\dfrac{1}{x}=\pm3$

5. α) $\alpha^2+4\ge4\alpha\Leftrightarrow\alpha^2-4\alpha+4\ge0\Leftrightarrow(\alpha-2)^2\ge0$ ισχύει

β) $x^2+16\ge8x\Leftrightarrow x^2-8x+16\ge0\Leftrightarrow(x-4)^2\ge0$ ισχύει

γ) $(\alpha+\beta)^2+4\alpha\beta\ge-8\beta^2\Leftrightarrow\alpha^2+2\alpha\beta+\beta^2+4\alpha\beta+8\beta^2\ge0\Leftrightarrow\alpha^2+6\alpha\beta+9\beta^2\ge0\Leftrightarrow(\alpha+3\beta)^2\ge0$ ισχύει

6. $x^2-2x+y^2-4y+5\le0\Leftrightarrow(x-1)^2+(y-2)^2\le0$, όμως $(x-1)^2+(y-2)^2\ge0$ άρα $(x-1)^2+(y-2)^2=0\Leftrightarrow$ $(x-1=0\Leftrightarrow x=1)$ ή $(y-2=0\Leftrightarrow y=2)$

7. α) $\alpha^3<\alpha\Leftrightarrow\alpha^3-\alpha<0\Leftrightarrow\alpha(\alpha-1)(\alpha+1)<0$ ισχύει αφού $0<\alpha<1$

β) $0<\alpha<1\Leftrightarrow0<\alpha^3<1$, $0<\alpha<1\Leftrightarrow\dfrac{1}{\alpha}>1$. Οπότε $0<\alpha^3<\alpha<1<\dfrac{1}{\alpha}$

8. α)

$$\begin{array}{r}2<x<3\\ 1\le y\le4\quad+\\ \hline 3<x+y<7\end{array}$$

β) $2<x<3\Leftrightarrow4<2x<6\ (1)$, $3\le y\le4\Leftrightarrow-9\ge-3y\ge-12\Leftrightarrow-12\le-3y\le-9\ (2)$

Από $(1)+(2)\Rightarrow4-12<2x-3y<6-9\Leftrightarrow-8<2x-3y<-3\Leftrightarrow-7<2x-3y+1<-2$

γ) $2<x<3\Leftrightarrow4<2x<6\ (1)$, $1\le y\le4\Leftrightarrow\dfrac{1}{1}\ge\dfrac{1}{y}\ge\dfrac{1}{4}\Leftrightarrow\dfrac{1}{4}\le\dfrac{1}{y}\le1\ (3)$

Από $(1)\cdot(3)\Rightarrow2\cdot\dfrac{1}{4}<2x\dfrac{1}{y}<6\cdot1\Leftrightarrow\dfrac{1}{2}<\dfrac{2x}{y}<6$

9. $\dfrac{\alpha^2+\beta^2}{\alpha+\beta}\ge\dfrac{\alpha+\beta}{2}\Leftrightarrow2(\alpha^2+\beta^2)\ge(\alpha+\beta)^2\Leftrightarrow2\alpha^2+2\beta^2\ge\alpha^2+2\alpha\beta+\beta^2\Leftrightarrow$

$2\alpha^2-\alpha^2-2\alpha\beta+2\beta^2-\beta^2\ge0\Leftrightarrow\alpha^2-2\alpha\beta+\beta^2\ge0\Leftrightarrow(\alpha-\beta)^2\ge0$ ισχύει

10. Είναι $-1<x<2\Leftrightarrow0<x+1<3$, $-1<x<2\Leftrightarrow-3<x-2<0$, $-1<x<2\Leftrightarrow1<x+2<4$,
$-1<x<2\Leftrightarrow-4<x-3<-1$, $-1<x<2\Leftrightarrow-2<2x<4\Leftrightarrow1<2x+3<7$ και
$-1<x<2\Leftrightarrow-4<4x<8\Leftrightarrow-13<4x-9<-1$, οπότε:
$A=|x+1|+|x-2|+|x+2|+|x-3|=x+1-x+2+x+2-x+3=8$,
$B=6|2x+3|+3|4x-9|=6(2x+3)+3(-4x+9)=12x+18-12x+27=45$

11. $A=\frac{|2x-1|}{|1-2x|}-3\frac{|3y+2|}{|-2-3y|}+2\frac{|x-y+z|}{|y-x-z|}=\frac{|2x-1|}{|-(2x-1)|}-3\frac{|3y+2|}{|-(3y+2)|}+2\frac{|x-y+z|}{|-(x-y+z)|}\Leftrightarrow$

$A=\frac{|2x-1|}{|2x-1|}-3\frac{|3y+2|}{|3y+2|}+2\frac{|x-y+z|}{|x-y+z|}=1-3+2=0$

12. Αν $x-2<0\Leftrightarrow x<2$ τότε $A=|x-2|+3x-5=-x+2+3x-5=2x-3$

Αν $x-2\geq 0\Leftrightarrow x\geq 2$ τότε $A=|x-2|+3x-5=x-2+3x-5=4x-7$

Αν $x-3<0\Leftrightarrow x<3$ τότε $B=|x-3|+2x-1=-x+3+2x-1=x+2$

Αν $x-3\geq 0\Leftrightarrow x\geq 3$ τότε $B=|x-3|+2x-1=x-3+2x-1=3x-4$

13. Πρέπει $|x|-3\neq 0\Leftrightarrow |x|\neq 3\Leftrightarrow x\neq \pm 3$, τότε: $A=\frac{x^2-9}{|x|-3}=\frac{\cancel{(|x|-3)}(|x|+3)}{\cancel{|x|-3}}=|x|+3$

14.

Απόλυτη τιμή	Απόσταση	Διάστημα ή ένωση διαστημάτων
$\|x-3\|<2$	$d(x,3)<2$	$(1,5)$
$\|x+2\|\leq 1$	$d(x,-2)<1$	$(-3,-1)$
$\|x-2\|>4$	$d(x,2)>4$	$(-\infty,-2)\cup(6,+\infty)$
$\|x-2\|\leq 2$	$d(x,2)\leq 2$	$[0,4]$

15. $\frac{\sqrt{3}}{\sqrt{3}-\sqrt{2}}-\frac{\sqrt{2}}{\sqrt{3}+\sqrt{2}}=\frac{\sqrt{3}(\sqrt{3}+\sqrt{2})-\sqrt{2}(\sqrt{3}-\sqrt{2})}{(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})}=\frac{(\sqrt{3})^2+\cancel{\sqrt{6}}-\cancel{\sqrt{6}}+(\sqrt{2})^2}{(\sqrt{3})^2-(\sqrt{2})^2}=\frac{3+2}{3-2}=5$

16. $A=\sqrt{2+\sqrt{2+\sqrt{3}}}\cdot\sqrt{2+\sqrt{3}}\cdot\sqrt{2-\sqrt{2+\sqrt{3}}}=\sqrt{\left(2+\sqrt{2+\sqrt{3}}\right)\left(2-\sqrt{2+\sqrt{3}}\right)\left(2+\sqrt{3}\right)}\Leftrightarrow$

$A=\sqrt{\left(2^2-\left(\sqrt{2+\sqrt{3}}\right)^2\right)\left(2+\sqrt{3}\right)}=\sqrt{\left(4-2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}=\sqrt{\left(2-\sqrt{3}\right)\left(2+\sqrt{3}\right)}\Leftrightarrow$

$A=\sqrt{2^2-\left(\sqrt{3}\right)^2}=\sqrt{4-3}=1$

17. A$=\sqrt{8}-\sqrt{12}-\sqrt{50}+\sqrt{75}=2\sqrt{2}-2\sqrt{3}-5\sqrt{2}+5\sqrt{3}=3\sqrt{3}-3\sqrt{2}$ και

$B=\sqrt{18}-\sqrt{27}-\sqrt{32}+\sqrt{48}=3\sqrt{2}-3\sqrt{3}-4\sqrt{2}+4\sqrt{3}=\sqrt{3}-\sqrt{2}$.

$A+B=3\sqrt{3}-3\sqrt{2}+\sqrt{3}-\sqrt{2}=4\sqrt{3}-4\sqrt{2}$,

$AB=\left(3\sqrt{3}-3\sqrt{2}\right)\left(\sqrt{3}-\sqrt{2}\right)=3\left(\sqrt{3}\right)^2-3\sqrt{6}-3\sqrt{6}+3\left(\sqrt{2}\right)^2=9-6\sqrt{6}+6=15-6\sqrt{6}$

$\frac{A}{B}=\frac{3\sqrt{3}-3\sqrt{2}}{\sqrt{3}-\sqrt{2}}=\frac{3\cancel{\left(\sqrt{3}-\sqrt{2}\right)}}{\cancel{\sqrt{3}-\sqrt{2}}}=3$

$\frac{1}{A}+\frac{1}{B}=\frac{1}{3\left(\sqrt{3}-\sqrt{2}\right)}+\frac{1}{\sqrt{3}-\sqrt{2}}=\frac{1+3}{3\left(\sqrt{3}-\sqrt{2}\right)}=\frac{4\left(\sqrt{3}+\sqrt{2}\right)}{3\left[\left(\sqrt{3}\right)^2-\left(\sqrt{2}\right)^2\right]}=\frac{4\left(\sqrt{3}+\sqrt{2}\right)}{3}$

18. $\dfrac{\sqrt{20}-2\sqrt{8}+3\sqrt{12}}{\sqrt{45}-2\sqrt{18}+3\sqrt{27}}=\dfrac{2\sqrt{5}-4\sqrt{2}+6\sqrt{3}}{3\sqrt{5}-6\sqrt{2}+9\sqrt{3}}=\dfrac{2\left(\sqrt{5}-2\sqrt{2}+3\sqrt{3}\right)}{3\left(\sqrt{5}-2\sqrt{2}+3\sqrt{3}\right)}=\dfrac{2}{3}$

19.α) $\sqrt[6]{3^5}\cdot\sqrt{3}\cdot\sqrt[3]{3}=3^{\frac{5}{6}}\cdot3^{\frac{1}{2}}\cdot3^{\frac{1}{3}}=3^{\frac{5}{6}+\frac{1}{2}+\frac{1}{3}}=3^{\frac{5}{6}+\frac{3}{6}+\frac{2}{6}}=3^{\frac{10}{6}}=3^{\frac{6}{6}+\frac{4}{6}}=3\cdot3^{\frac{2}{3}}=3\sqrt[3]{9}$

β) $\sqrt[4]{2^5}\cdot\sqrt[12]{2^9}=2^{\frac{5}{4}}\cdot2^{\frac{9}{12}}=2^{\frac{5}{4}+\frac{9}{12}}=2^{\frac{15}{12}+\frac{9}{12}}=2^{\frac{24}{12}}=2^2=4$

20. $\left(\sqrt{2}-1\right)^3=\left(\sqrt{2}\right)^3-3\left(\sqrt{2}\right)^2\cdot1+3\sqrt{2}\cdot1^2-1^3=2\sqrt{2}-6+3\sqrt{2}-1=5\sqrt{2}-7$ και

$\left(\sqrt{2}+1\right)^3=\left(\sqrt{2}\right)^3+3\left(\sqrt{2}\right)^2\cdot1+3\sqrt{2}\cdot1^2+1^3=2\sqrt{2}+6+3\sqrt{2}+1=5\sqrt{2}+7$,

$A=\sqrt[3]{5\sqrt{2}-7}-\sqrt[3]{7+5\sqrt{2}}=\sqrt[3]{\left(\sqrt{2}-1\right)^3}-\sqrt[3]{\left(\sqrt{2}+1\right)^3}=\sqrt{2}-1-\left(\sqrt{2}+1\right)=-2$.

21.α) $|x|=6\Leftrightarrow x=\pm6$

β) $|x-1|=3\Leftrightarrow\left(x-1=3\Leftrightarrow x=4\right)$ ή $\left(x-1=-3\Leftrightarrow x=-2\right)$

γ) $\left|3-|x+2|\right|=1\Leftrightarrow\left(3-|x+2|=1\Leftrightarrow 2=|x+2|\right)$ ή $\left(3-|x+2|=-1\Leftrightarrow 4=|x+2|\right)$

$|x+2|=2\Leftrightarrow\left(x+2=2\Leftrightarrow x=0\right)$ ή $\left(x+2=-2\Leftrightarrow x=-4\right)$ ή

$|x+2|=4\Leftrightarrow\left(x+2=4\Leftrightarrow=2\right)$ ή $\left(x+2=-4\Leftrightarrow x=-6\right)$

δ) $|2x-3|=|x+6|\Leftrightarrow\left(2x-3=x+6\Leftrightarrow 2x-x=6+3\Leftrightarrow x=9\right)$ ή

$\left(2x-3=-x-6\Leftrightarrow 2x+x=-6+3\Leftrightarrow 3x=-3\Leftrightarrow x=-1\right)$

ε) $\dfrac{|x|-3}{6}-\dfrac{|x|-2}{4}=-1\Leftrightarrow 12\dfrac{|x|-3}{6}-12\dfrac{|x|-2}{4}=-12\Leftrightarrow 2|x|-6-3|x|+6=-12\Leftrightarrow -|x|=-12\Leftrightarrow$

$|x|=12\Leftrightarrow x=\pm12$

στ) $\dfrac{3|x-1|-1}{2}-2=\dfrac{|x-1|-4}{3}\Leftrightarrow 6\dfrac{3|x-1|-1}{2}-12=6\dfrac{|x-1|-4}{3}\Leftrightarrow 9|x-1|-3-12=2|x-1|-8\Leftrightarrow$

$9|x-1|-2|x-1|=-8+3+12\Leftrightarrow 7|x-1|=7\Leftrightarrow|x-1|=1\Leftrightarrow\left(x-1=1\Leftrightarrow x=2\right)$ ή $\left(x-1=-1\Leftrightarrow x=0\right)$

ζ) $|3x-2|=x+1$

Αν $x+1\geq0\Leftrightarrow x\geq-1$ τότε η εξίσωση γίνεται:

$\left(3x-2=x+1\Leftrightarrow 3x-x=1+2\Leftrightarrow 2x=3\Leftrightarrow x=\dfrac{3}{2}\text{ δεκτή}\right)$ ή

$\left(3x-2=-x-1\Leftrightarrow 3x+x=-1+2\Leftrightarrow 4x=1\Leftrightarrow x=\dfrac{1}{4}\text{ δεκτή}\right)$

η) $\lambda x=\lambda+2$

Αν $\lambda\neq0$ τότε $x=\dfrac{\lambda+2}{\lambda}$ και αν $\lambda=0$ τότε $0x=2$ αδύνατη

θ) $\left(\lambda-2\right)x=\lambda^2-4$

Αν $\lambda-2\neq0\Leftrightarrow\lambda\neq2$ τότε $x=\dfrac{\left(\lambda-2\right)\left(\lambda+2\right)}{\lambda-2}=\lambda+2$ και

αν $\lambda=2$ τότε $0x=0$ ταυτότητα

22.α) $A \ge 4 \Leftrightarrow \alpha + \frac{4}{\alpha} \ge 4 \Leftrightarrow \alpha \cdot \alpha + \cancel{\alpha} \cdot \frac{4}{\cancel{\alpha}} \ge 4 \cdot \alpha \Leftrightarrow \alpha^2 + 4 \ge 4\alpha \Leftrightarrow \alpha^2 - 4\alpha + 4 \ge 0 \Leftrightarrow (\alpha - 2)^2 \ge 0$ ισχύει

β) Είναι $\alpha + \frac{4}{\alpha} \ge 4 \Leftrightarrow \frac{\alpha^2 + 4}{\alpha} \ge 4$ και $\frac{\beta^2 + 4}{\beta} \ge 4$, οπότε με πολλαπλασιασμό κατά μέλη έχουμε:

$$\frac{\alpha^2 + 4}{\alpha} \cdot \frac{\beta^2 + 4}{\beta} \ge 4 \cdot 4 \Leftrightarrow \frac{(\alpha^2 + 4)(\beta^2 + 4)}{\alpha\beta} \ge 16$$

γ) $\alpha \in (4,8) \Leftrightarrow 4 < \alpha < 8$ (1) $\Leftrightarrow \frac{1}{4} > \frac{1}{\alpha} > \frac{1}{8} \Leftrightarrow \frac{1}{8} < \frac{1}{\alpha} < \frac{1}{4} \Leftrightarrow \cancel{4} \cdot \frac{1}{\cancel{8}^2} < 4 \cdot \frac{1}{\alpha} < \cancel{4} \cdot \frac{1}{\cancel{4}} \Leftrightarrow \frac{1}{2} < \frac{4}{\alpha} < 1$ (2).

Με πρόσθεση κατά μέλη των (1),(2) έχουμε: $4 + \frac{1}{2} < \alpha + \frac{4}{\alpha} < 8 + 1 \Leftrightarrow 4,5 < A < 9$

δ) $A = 4 \Leftrightarrow (\alpha - 2)^2 = 0 \Leftrightarrow \alpha = 2$

Για κάθε $x > 2$ είναι $x + 1 > 3 \Rightarrow x + 1 > 0$ και $x > 2 \Leftrightarrow 0 > 2 - x$, άρα

$|x + 1| - |2 - x| = x + 1 - (-2 + x) = \cancel{x} + 1 + 2 - \cancel{x} = 3$

ε) i. $A = \sqrt{5} - 1 + \frac{4}{\sqrt{5} - 1} = \sqrt{5} - 1 + \frac{4(\sqrt{5} + 1)}{(\sqrt{5} - 1)(\sqrt{5} + 1)} = \sqrt{5} - 1 + \frac{4(\sqrt{5} + 1)}{(\sqrt{5})^2 - 1^2} = \sqrt{5} - 1 + \frac{4(\sqrt{5} + 1)}{5 - 1} \Leftrightarrow$

$$A = \sqrt{5} - 1 + \frac{\cancel{4}(\sqrt{5} + 1)}{\cancel{4}} = \sqrt{5} - \cancel{1} + \sqrt{5} + \cancel{1} = 2\sqrt{5}$$

ii. $\sqrt[3]{A} \cdot \sqrt[6]{2500} \cdot \sqrt[12]{400} = \sqrt[3]{2\sqrt{5}} \cdot \sqrt[6]{5^4 \cdot 2^2} \cdot \sqrt[12]{2^4 \cdot 5^2} = \sqrt[3]{2 \cdot 5^{\frac{1}{2}}} \cdot \sqrt[6]{5^4} \cdot \sqrt[6]{2^2} \cdot \sqrt[12]{2^4} \cdot \sqrt[12]{5^2} =$

$$\sqrt[3]{2} \cdot \sqrt[3]{5^{\frac{1}{2}}} \cdot 5^{\frac{4}{6}} \cdot 2^{\frac{2}{6}} \cdot 2^{\frac{4}{12}} \cdot 5^{\frac{2}{12}} = 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} \cdot 2^{\frac{1}{3}} \cdot 5^{\frac{\frac{1}{2}}{3}} \cdot 5^{\frac{2}{3}} \cdot 5^{\frac{1}{6}} = 2^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} \cdot 5^{\frac{1}{6}+\frac{2}{3}+\frac{1}{6}} = 2^{\frac{3}{3}} \cdot 5^{\frac{1}{6}+\frac{4}{6}+\frac{1}{6}}$$

$$2 \cdot 5^{\frac{6}{6}} = 2 \cdot 5 = 10$$

ή $\sqrt[3]{A} \cdot \sqrt[6]{2500} \cdot \sqrt[12]{400} = \sqrt[3]{2\sqrt{5}} \cdot \sqrt[6]{2500} \cdot \sqrt[\cancel{12}\,6]{20^{\cancel{2}}} = \sqrt[3]{\sqrt{2^2 \cdot 5}} \cdot \sqrt[6]{2500 \cdot 20} =$

$$\sqrt[6]{4 \cdot 5} \cdot \sqrt[6]{50000} = \sqrt[6]{4 \cdot 5 \cdot 50000} = \sqrt[6]{1000000} = 10$$

Στέλιος Μιχαήλογλου