

### 4.3 Ανισώσεις Γινόμενο & Ανισώσεις Πηλίκο

Δευτέρα, 8 Μαρτίου 2021 5:08 μμ

$$P(x) = x(x-1)(x-2)(x-3) = x^4 + \dots$$

4<sup>ου</sup> βαθμού πολυώνυμο

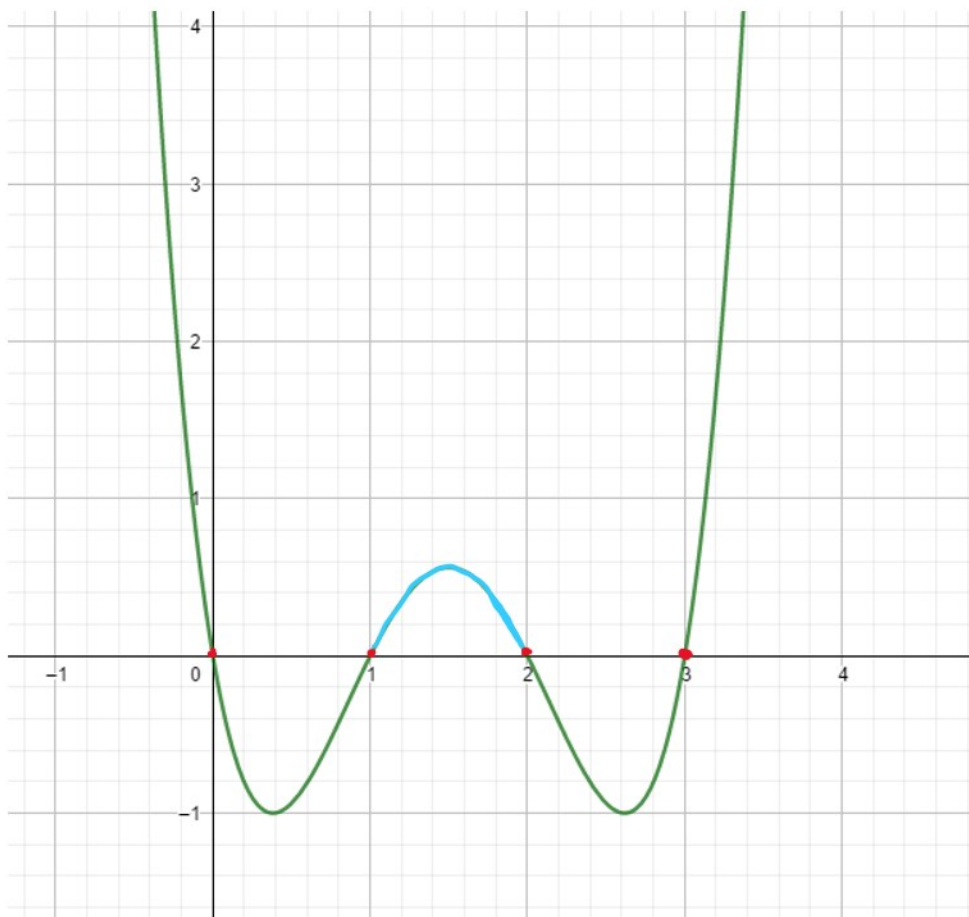
$$P(x) = 0 \Rightarrow x=0 \text{ ή } x-1=0 \text{ ή } x-2=0 \text{ ή } x-3=0 \Rightarrow$$

$$x=0 \text{ ή } x=1 \text{ ή } x=2 \text{ ή } x=3$$

|      |           |   |   |   |   |           |
|------|-----------|---|---|---|---|-----------|
| x    | $-\infty$ | 0 | 1 | 2 | 3 | $+\infty$ |
| P(x) | +         | 0 | - | 0 | - | +         |

↑  
περίστροφ του  
μεγιστογραφικού  
σημείου

$$P(x) \geq 0 \Rightarrow x \in (-\infty, 0] \cup [1, 2] \cup [3, +\infty)$$



$$P(x) = (x-1) (x^2 - 4x + 3)$$

$$= (x-1) (x-1) (x-3)$$

$$= (x-1)^{\textcircled{2}} (x-3)$$

↑  
≥ 0

Βρίσκω τα σημεία των παλινδρόμων

| X         | $-\infty$ | 1 | 3 | $+\infty$ |
|-----------|-----------|---|---|-----------|
| $(x-1)^2$ | +         | 0 | + | +         |
| $x-3$     | -         | - | 0 | +         |
|           |           | + | + | +         |

Πολλαπλαιάζω  
τις τιμές  
τε του κανόνα  
το - προσήλου

|        |  |   |  |   |  |   |
|--------|--|---|--|---|--|---|
| $x-3$  |  | - |  | - |  |   |
| $p(x)$ |  | - |  | 0 |  | + |

to - position

Aufgabe 1  $\in \mathbb{R}$ . 117

$$p(x) = (2-3x)(x^2-x-2)(x^2-x+1)$$

$$p(x) = 0 \Rightarrow$$

$$2-3x = 0 \quad \text{in} \quad x^2-x-2=0 \quad \text{in} \quad x^2-x+1=0$$

$$x = \frac{2}{3}$$

$$\Delta = 1+8=9 \quad \text{in}$$

$$x = \frac{1 \pm \sqrt{9}}{2} =$$

$$x = \frac{1 \pm 3}{2} < \frac{1}{2}$$

$$\Delta = 1-4 = -3 < 0$$

(Aussagen)

Diskriminante negativ

$$x^2-x+1 > 0 \quad \text{für alle} \quad x \in \mathbb{R}$$

|           |           |               |   |   |           |
|-----------|-----------|---------------|---|---|-----------|
| $x$       | $-\infty$ | $\frac{2}{3}$ | 1 | 2 | $+\infty$ |
| $2-3x$    | +         | 0             | - | - | -         |
| $x^2-x-2$ | +         | +             | 0 | - | +         |
| $x^2-x+1$ | +         | +             | + | + | +         |
| $p(x)$    | +         | 0             | - | + | -         |

$$\begin{aligned} + \cdot + &= + \\ + \cdot - &= - \\ - \cdot + &= - \\ - \cdot - &= + \end{aligned}$$

Aufgabe 2  $\in \mathbb{R}$ . 117

$$p(x) = (-x^2+4)(x^2-3x+2)(x^2+x+1)$$

$$= (x-1)(x-2)(x^2+x+1)$$

$$\begin{aligned}
 p(x) &= (-x+4)^4 \\
 &= (2-x)(2+x)(x-1)(x-2)(x^2+x+1) \\
 &= -(x-2)^2(x+2)(x-1)(x^2+x+1) \\
 &= (x-2)^2(x+2)(1-x)(x^2+x+1)
 \end{aligned}$$

↓  
≥ 0

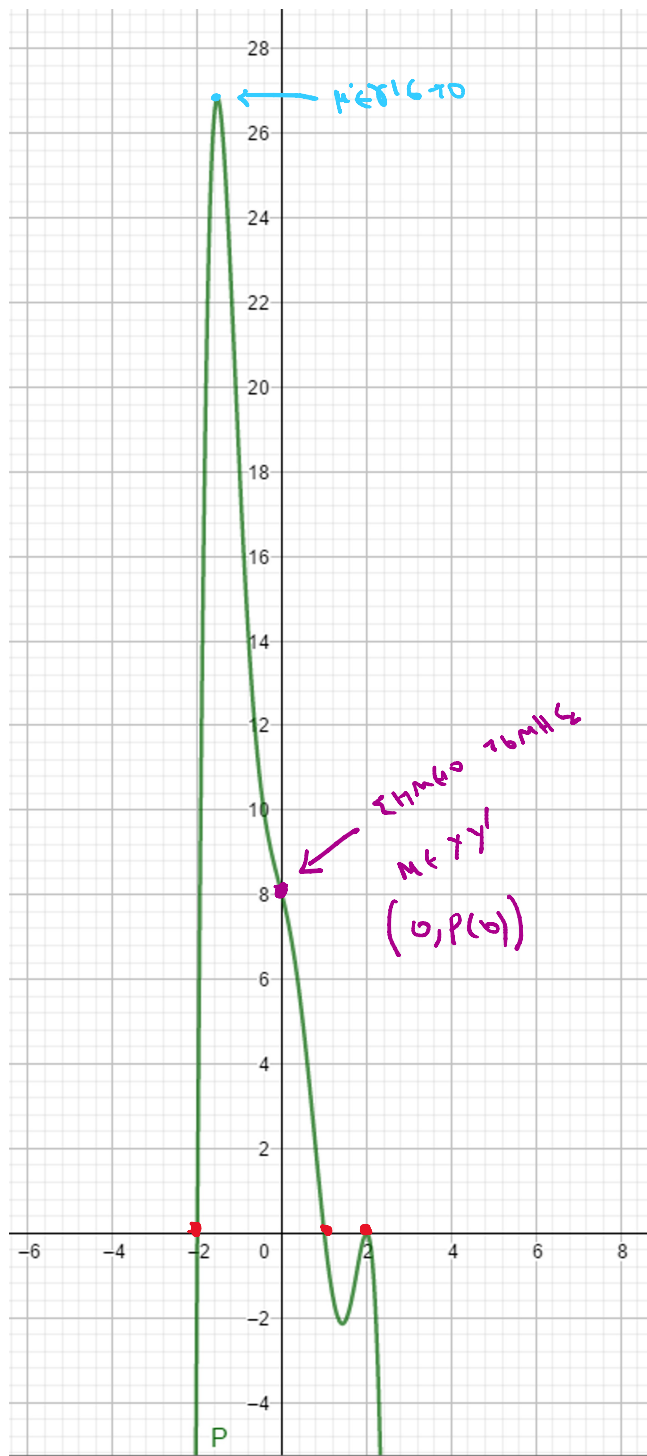
↓  
 $\Delta = 1 - 4 = -3 < 0$

·  $A(x) = x^2 + x + 1 > 0$

| x         | $-\infty$ | -2 | 1 | 2 | $+\infty$ |
|-----------|-----------|----|---|---|-----------|
| $(x-2)^2$ | +         | +  | + | + | +         |
| $x+2$     | -         | 0  | + | + | +         |
| $1-x$     | +         | +  | + | 0 | -         |
| $x^2+x+1$ | +         | +  | + | + | +         |
| $p(x)$    | -         | 0  | + | 0 | -         |

← προσοχή

Επίσης παρατηρείται:



ΣΗΜΕΙΑ ΤΟΜΗΣ  
ΜΕ ΧΧ'  
ΛΟΓΕΙΣ ΤΗ  
ΕΤΙΩΣΗ  $P(x)=0$

$$P(0) = (-0^2 + 4)(0^2 - 3 \cdot 0 + 2)(0^2 + 0 + 1)$$

$$= 4 \cdot 2 \cdot 1 = 8$$

Άσκηση 3 α). 117

$$P(x) = (x-1)(x^2+2)(x^2-9)$$

1 1 2 2 1

$$\begin{aligned}
 p(x) &= (x-1)(x^2+2)(x^2-9) \\
 &= (x-1)(x^2+2)(x-3)(x+3)
 \end{aligned}$$

↓  
 $> 0$

| X    | $-\infty$ | -3 | 1 | 3 | $+\infty$ |
|------|-----------|----|---|---|-----------|
| p(x) | -         | +  | - | + | +         |

$$p(x) > 0 \Rightarrow x \in (-3, 1) \cup (3, +\infty)$$

Aufgabe 4 6.11.17

$$\begin{aligned}
 p(x) &= (3-x)(2x^2+6x)(x^2+3) \\
 &= (3-x) \cdot 2x \cdot (x+3) \cdot (x^2+3)
 \end{aligned}$$

↓  
 $> 0$

$$p(x) = 0 \Rightarrow x = 3 \text{ in } x = 0 \text{ in } x = -3$$

| x   | $-\infty$ | -3 | 0 | 3 | $+\infty$ |
|-----|-----------|----|---|---|-----------|
| 3-x | +         | +  | + | 0 | -         |
| 2x  | -         | -  | 0 | + | +         |
| x+3 | -         | 0  | + | + | +         |

|         |   |   |   |   |   |
|---------|---|---|---|---|---|
| $x+3$   | - | 0 | + | + | + |
| $x^2+3$ | + | + | + | + | + |
| $p(x)$  | + | 0 | - | 0 | + |

$$p(x) < 0 \Rightarrow -3 < x < 0 \quad \text{in} \quad x > 3$$

$$x \in (-3, 0) \cup (3, +\infty)$$

Aufgabe 5 c) 117

$$p(x) = (2 - x - x^2)(x^2 + 2x + 1)$$

$$(-x^2 - x + 2)(x+1)^2$$

$$\Delta = 1 + 8 = 9$$

$$x = \frac{1 \pm 3}{-2} = \frac{-1 \pm 3}{-2} \begin{matrix} 1 \\ -2 \end{matrix}$$

|                |           |      |      |     |           |
|----------------|-----------|------|------|-----|-----------|
| $x$            | $-\infty$ | $-2$ | $-1$ | $1$ | $+\infty$ |
| $-x^2 - x + 2$ | -         | 0    | +    | +   | 0         |
| $(x+1)^2$      | +         | +    | +    | 0   | +         |
| $p(x)$         | -         | 0    | +    | 0   | +         |

$$\Rightarrow x \in (-\infty; 2] \cup [2; +\infty)$$

$$p(x) \leq 0 \Rightarrow x \in (-\infty; 2] \cup [2, \dots)$$

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$$p(x) = (x-3)(2x^2+x-3)(x-1-2x^2)$$

$$x=3$$

$$x = \frac{3}{2}$$

$$x=1$$

$$-2x^2+x-1$$

$$\Delta = 1 - 4(-2)(-1)$$

$$= 1 + 8 = 9$$

$$x = \frac{-1 \pm 3}{-2} = \begin{matrix} 2 \\ -1 \end{matrix}$$

| x           | $-\infty$ | -1 | 1 | $\frac{3}{2}$ | 2 | 3 | $+\infty$ |
|-------------|-----------|----|---|---------------|---|---|-----------|
| $x-3$       | -         | -  | - | -             | - | - | +         |
| $2x^2+x-3$  | +         | +  | + | -             | + | + | +         |
| $-2x^2+x-1$ | -         | -  | + | +             | + | - | -         |
| $p(x)$      | +         | -  | - | +             | - | + | -         |

$$p(x) > 0 \Rightarrow x \in (-\infty, -1) \cup (1, \frac{3}{2}) \cup (2, 3)$$