2.3 Ρίζες ασκήσεις

Κυριακή, 13 Δεκεμβρίου 2020 10:45 πμ

Θυμίζω ότι αν $a > 0$

$$\sqrt[\nu]{a^{\mu}} = a^{\frac{\mu}{\nu}}$$

π.χ. $\sqrt[5]{a^{3}} = a^{\frac{3}{5}}$

$$\sqrt{a^{7}} = a^{\frac{7}{2}}$$

εδώ εννοείται ότι υπάρχει το 2
το οποίο παραλείπεται

$$\sqrt[3]{a} = a^{\frac{1}{3}}$$

εδώ παραλείπεται το 1

$$\boxed{a^{\mu} \cdot a^{\nu} = a^{\mu+\nu}}$$

Άσκηση 7 σελ. 74

1) $\sqrt{\sqrt{2 \cdot \sqrt[3]{2}}} = \sqrt{\sqrt{2 \cdot 2^{1/3}}} =$

i) $\sqrt{\sqrt{2}\cdot\sqrt[3]{2}}$

$= \left(\left(2^{1}\cdot 2^{1/3}\right)^{1/2}\right)^{1/2} =$

$\left(\left(2^{1+\frac{1}{3}}\right)^{1/2}\right)^{1/2} = \left(\left(2^{4/3}\right)^{1/2}\right)^{1/2}$

$= 2^{\frac{4}{3}\cdot\frac{1}{2}\cdot\frac{1}{2}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$

ii) $\sqrt[5]{2\sqrt{2\sqrt[3]{2}}} = \left(2\left(2\cdot 2^{\frac{1}{3}}\right)^{\frac{1}{2}}\right)^{\frac{1}{5}} =$

$\left(2\left(2^{1+\frac{1}{3}}\right)^{\frac{1}{2}}\right)^{\frac{1}{5}} =$

$\left(2\left(2^{\frac{4}{3}}\right)^{\frac{1}{2}}\right)^{\frac{1}{5}} =$

$\left(\ \ \frac{4}{3}\cdot\frac{1}{2}\right)^{\frac{1}{5}}$

$\frac{1}{1}+\frac{1}{3} =$

$\frac{3}{3}+\frac{1}{3} = \frac{4}{3}$

2

$$\left(2 \cdot 2^{\frac{4}{3}\cdot\frac{1}{2}}\right)^{5} =$$

$$\frac{\cancel{4}^{\,2}}{3}\cdot\frac{1}{\cancel{2}} = \frac{2}{3}$$

$$\left(2^{1} \cdot 2^{\frac{2}{3}}\right)^{\frac{1}{5}} =$$

$$\frac{\overset{3}{1}}{1} + \frac{\overset{1}{2}}{3} = \frac{3}{3} + \frac{2}{3}$$

$$= \frac{5}{3}$$

$$\left(2^{1+\frac{2}{3}}\right)^{\frac{1}{5}} =$$

$$\left(2^{\frac{5}{3}}\right)^{\frac{1}{5}} =$$

$$2^{\frac{\cancel{5}}{3}\cdot\frac{1}{\cancel{5}}} = 2^{\frac{1}{3}} = \sqrt[3]{2}$$

Άσκηση 8 σελ. 74

i) $\sqrt[4]{3^{3}} \cdot \sqrt[3]{3} = 3 \cdot \sqrt[12]{3}$

Διαδοχικά:

$$3^{\frac{3}{4}} \cdot 3^{\frac{1}{3}} = 3 \cdot 3^{\frac{1}{12}}$$

$$3^{\frac{3}{4}} \cdot 3^{\frac{1}{3}} = 3 \cdot 3^{\frac{1}{12}}$$

$$3^{\frac{3}{4}+\frac{1}{3}} = 3^{1+\frac{1}{12}}$$

$$\frac{3}{4}+\frac{1}{3} = \frac{9}{12}+\frac{4}{12} = \frac{13}{12}$$

Εκπ(3,4)=12

$$\frac{1}{1}+\frac{1}{12} = \frac{12}{12}+\frac{1}{12} = \frac{13}{12}$$

Εκπ(1,12)=12

$$3^{\frac{13}{12}} = 3^{\frac{13}{12}}$$ Ισχύει

ii) $\sqrt[9]{2^8} \cdot \sqrt[6]{2^5} = 2 \cdot \sqrt[18]{2^{13}}$

Διαδοχικά:

$$\frac{8}{\ } \qquad \frac{5}{\ } \qquad 1 \qquad \frac{13}{18}$$

$$2^{\frac{8}{9}} \cdot 2^{\frac{5}{6}} = 2^{1} \cdot 2^{\frac{}{18}}$$

$$2^{\frac{8}{9}+\frac{5}{6}} = 2^{1+\frac{13}{18}}$$

$$\overset{2)}{\frac{8}{9}} + \overset{3)}{\frac{5}{6}} = \frac{16}{18} + \frac{15}{18} = \frac{31}{18}$$

ΕΚΠ $(9,6) = 18$

$$\overset{18)}{\frac{1}{1}} + \overset{1)}{\frac{13}{18}} = \frac{18}{18} + \frac{13}{18} = \frac{31}{18}$$

$$2^{\frac{31}{18}} = 2^{\frac{31}{18}} \quad \text{ισχύει}$$

iii) $$\sqrt{5^{3}} \cdot \sqrt[3]{5} \cdot \sqrt[6]{5^{4}} = 25 \cdot \sqrt{5}$$

Διαδοχικά:

Διαδοχικά:

$$5^{\frac{3}{2}} \cdot 5^{\frac{1}{3}} \cdot 5^{\frac{4}{6}} = 5^{2} \cdot 5^{\frac{1}{2}}$$

$$5^{\frac{3}{2}+\frac{1}{3}+\frac{4}{6}} = 5^{2+\frac{1}{2}}$$

$$\overset{3}{\frac{3}{2}} + \overset{2}{\frac{1}{3}} + \overset{1}{\frac{4}{6}} = \frac{9}{6} + \frac{2}{6} + \frac{4}{6} = \frac{15}{6} = \frac{5}{2}$$

απλοποίηση με το 3

ΕΚΠ(2,3,6)

$$\overset{2}{\frac{2}{1}} + \overset{1}{\frac{1}{2}} = \frac{4}{2} + \frac{1}{2} = \frac{5}{2}$$

$$5^{\frac{5}{2}} = 5^{\frac{5}{2}} \quad \text{ισχύει}$$

Άσκηση 9 σελ. 74

$$25 \cdot \sqrt{12} \quad - 10$$

i) $$\frac{25\cdot\sqrt{12}}{\sqrt{75}} = 10$$

Ξεκινάω από αριστερά να καταλήξω δεξιά

$$\frac{25\cdot\sqrt{12}}{\sqrt{75}} = \frac{5^2\cdot\sqrt{3\cdot 4}}{\sqrt{3\cdot 25}} =$$

αν $\alpha,\beta>0$:

$$\sqrt[\nu]{\alpha\beta} = \sqrt[\nu]{\alpha}\cdot\sqrt[\nu]{\beta}$$

$$= \frac{5^2\cdot\sqrt{3}\cdot\sqrt{4}}{\sqrt{3}\cdot\sqrt{25}}$$

$$= \frac{5^{\cancel{2}}\cdot\cancel{\sqrt{3}}\cdot 2}{\cancel{\sqrt{3}}\cdot\cancel{5}} =$$

$$= 5\cdot 2 = 10$$

ii) $$\frac{\sqrt{216}\cdot\sqrt{75}}{\sqrt{50}} = 18$$

Ξεκινάω από αριστερά να καταλήξω δεξιά

$$\sqrt{216}\cdot\sqrt{75}$$

$$\frac{\sqrt{216}\cdot\sqrt{75}}{\sqrt{50}} =$$

Ανάλυση του 216 σε πρώτους

$$\begin{array}{r|l}
216 & 2 \\
108 & 2 \\
54 & 2 \\
27 & 3 \\
9 & 3 \\
3 & 3 \\
1 &
\end{array}$$

3 δυάρια

3 τριάρια

$$2^3 \cdot 3^3$$

$$= \frac{\sqrt{2^3\cdot 3^3}\cdot\sqrt{3\cdot 5^2}}{\sqrt{2\cdot 5^2}} =$$

$$= \frac{\sqrt{2^3}\cdot\sqrt{3^3}\cdot\sqrt{3}\cdot\cancel{\sqrt{5^2}}}{\sqrt{2}\cdot\cancel{\sqrt{5^2}}} =$$

$$= \frac{\sqrt{2^2}\cdot\cancel{\sqrt{2}}\cdot\sqrt{3^2}\cdot\sqrt{3}\cdot\sqrt{3}}{\cancel{\sqrt{2}}}$$

$$= 2\cdot 3\cdot 3 = 18$$

Άσκηση 10 σελ. 75

Θυμίζω

$$(a-\beta)(a+\beta) = a^2-\beta^2$$

Γινόμενο διαφοράς με άθροισμα
καταλήγει σε διαφορά τετραγώνων

π.χ.

$$(x-2)(x+2) = x^2-2^2 = x^2-4$$

$$(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y}) = (\sqrt{x})^2-(\sqrt{y})^2 = x-y$$

$$\frac{4}{\ } = \frac{4(5+\sqrt{3})}{\ } =$$

$$\frac{4}{5-\sqrt{3}} = \frac{}{(5-\sqrt{3})(5+\sqrt{3})} =$$

$$\frac{4(5+\sqrt{3})}{5^2-(\sqrt{3})^2} = \frac{4(5+\sqrt{3})}{25-3} =$$

$$\frac{\cancel{4}^{\,2}(5+\sqrt{3})}{\cancel{22}_{\,11}} = \frac{2(5+\sqrt{3})}{11} = \frac{10+2\sqrt{3}}{11}$$

ii) $$\frac{8}{\sqrt{7}-\sqrt{5}} = \frac{8(\sqrt{7}+\sqrt{5})}{\underbrace{(\sqrt{7}-\sqrt{5})(\sqrt{7}+\sqrt{5})}} =$$

$$\frac{8(\sqrt{7}+\sqrt{5})}{\sqrt{7}^2-\sqrt{5}^2} = \frac{8(\sqrt{7}+\sqrt{5})}{7-5} =$$

$$\frac{\cancel{8}^{\,4}(\sqrt{7}+\sqrt{5})}{\cancel{2}_{\,1}} = 4(\sqrt{7}+\sqrt{5})$$

iii) $\sqrt{7}+\sqrt{6}$ $\quad (\sqrt{7}+\sqrt{6})(\sqrt{7}+\sqrt{6}) =$

iii) $$\frac{\sqrt{7}+\sqrt{6}}{\sqrt{7}-\sqrt{6}} = \frac{(\sqrt{7}+\sqrt{6})(\sqrt{7}+\sqrt{6})}{(\sqrt{7}-\sqrt{6})(\sqrt{7}+\sqrt{6})} =$$

$$\frac{(\sqrt{7}+\sqrt{6})^2}{\sqrt{7}^2-\sqrt{6}^2} = \frac{(\sqrt{7}+\sqrt{6})^2}{\underset{1}{\cancel{7-6}}} = (\sqrt{7}+\sqrt{6})^2$$

Άσκηση 11 σελ. 75

i) Ανάλυση των 162, 98, 50, 32

σε γινόμενο πρώτων παραγόντων

162	2
81	3
27	3
9	3
3	3
1	

$162 = 2 \cdot 3^4$

98	2
49	7
7	7
1	

$98 = 2 \cdot 7^2$

32	2
16	2
	2

50	2

$$\begin{array}{r|l} 50 & 2 \\ 25 & 5 \\ 5 & 5 \\ 1 & \end{array} \qquad 50 = 2 \cdot 5^2$$

$$\begin{array}{r|l} 16 & 2 \\ 8 & 2 \\ 4 & 2 \\ 2 & 2 \\ 1 & \end{array} \qquad 32 = 2^5$$

$$\frac{\sqrt{162} + \sqrt{98}}{\sqrt{50} - \sqrt{32}} = \frac{\sqrt{2 \cdot 3^4} + \sqrt{2 \cdot 7^2}}{\sqrt{2 \cdot 5^2} + \underbrace{\sqrt{2^5}}_{\sqrt{2 \cdot 2^4}}} =$$

$$= \frac{\sqrt{2} \cdot \sqrt{3^4} + \sqrt{2}\,\sqrt{7^2}}{\sqrt{2} \cdot \sqrt{5^2} - \sqrt{2} \cdot \sqrt{2^4}} =$$

$$= \frac{\cancel{\sqrt{2}}\left(\sqrt{3^4} + \sqrt{7^2}\right)}{\cancel{\sqrt{2}}\left(\sqrt{5^2} - \sqrt{2^4}\right)} =$$

$$= \frac{3^{\frac{4}{2}} + 7}{5 - 2^{4/2}} =$$

$$= \frac{3^2+7}{5-2^2} = \frac{9+7}{5-4} = \frac{16}{1} = 16$$

ii) $9 = 3^2$, $27 = 3^3$

$$\sqrt{\frac{9^{12}+3^{20}}{9^{11}+27^{6}}} = \left(\frac{(3^2)^{12}+3^{20}}{(3^2)^{11}+(3^3)^6}\right)^{\frac{1}{2}} =$$

$$\left(\frac{3^{2\cdot 12}+3^{20}}{3^{2\cdot 11}+3^{3\cdot 6}}\right)^{\frac{1}{2}} =$$

$$\left(\frac{3^{24}+3^{20}}{3^{22}+3^{18}}\right)^{\frac{1}{2}} =$$

$\frac{1}{2}$

$$\left(\frac{3^{20}\cancel{(3^4+1)}}{3^{18}\cancel{(3^4+1)}}\right) =$$

$$\left(3^{20-18}\right)^{\frac{1}{2}} = \left(3^2\right)^{\frac{1}{2}} = 3$$