

2.15

ii) $\lim_{x \rightarrow +\infty} (3x + \sigma \nu \chi)$

$$-1 \leq \sigma \nu \chi \leq 1$$

$$\Rightarrow 3x - 1 \leq 3x + \sigma \nu \chi \leq 3x + 1$$

$$\left. \begin{array}{l} \lim_{x \rightarrow +\infty} (3x - 1) = +\infty \\ \lim_{x \rightarrow +\infty} (3x + 1) = +\infty \end{array} \right\} \lim_{x \rightarrow +\infty} (3x + \sigma \nu \chi) = +\infty$$

iii) $\lim_{x \rightarrow -\infty} \left(\frac{x^5 + 4x + 1}{x^6 + 2x - 5} \cdot \eta \mu 4x \right)$

$$\lim_{x \rightarrow -\infty} \frac{x^5 + 4x + 1}{x^6 + 2x - 5} = \lim_{x \rightarrow -\infty} \frac{x^5}{x^6} = \lim_{x \rightarrow -\infty} \frac{1}{x} = 0$$

$$-1 \leq \eta \mu 4x \leq 1$$

$$\Rightarrow |\eta \mu 4x| \leq 1$$

$$\Rightarrow \left| \frac{x^5 + 4x + 1}{x^6 + 2x - 5} \cdot \eta \mu 4x \right| \leq \left| \frac{x^5 + 4x + 1}{x^6 + 2x - 5} \right|$$

$$\Rightarrow - \left| \frac{x^5 + 4x + 1}{x^6 + 2x - 5} \right| \leq \frac{x^5 + 4x + 1}{x^6 + 2x - 5} \eta \mu 4x \leq \left| \frac{x^5 + 4x + 1}{x^6 + 2x - 5} \right|$$

Με κριτήριο παρεμβολής $\lim_{x \rightarrow -\infty} \frac{x^5 + 4x + 1}{x^6 + 2x - 5} \eta \mu 4x = 0$

iv) $\lim_{x \rightarrow +\infty} \left(\frac{2x + 3}{\sqrt{x^2 + 1} + x} \cdot \sigma \nu \nu \frac{2x + 1}{x^2 + 5} \right) = 1 \cdot 1 = 1$

$$\cdot \lim_{x \rightarrow +\infty} \frac{2x + 3}{\sqrt{x^2 + 1} + x} = \lim_{x \rightarrow +\infty} \frac{2x + 3}{\sqrt{x^2(1 + \frac{1}{x^2})} + x}$$

$$\stackrel{x > 0}{=} \lim_{x \rightarrow +\infty} \frac{x(2 + \frac{3}{x})}{x(\sqrt{1 + \frac{1}{x^2}} + 1)}$$

$$= \frac{2+0}{\sqrt{1+0}+1} = \frac{2}{2} = 1$$

$$\bullet \lim_{x \rightarrow +\infty} \text{GUV} \frac{2x+1}{x^2+5} = \lim_{u \rightarrow 0} \text{GUV} u = 1$$

$$\text{Θ \acute{E} TW} \quad u = \frac{2x+1}{x^2+5} \quad \lim_{x \rightarrow +\infty} u = \lim_{x \rightarrow +\infty} \frac{2x}{x^2} =$$

$$= \lim_{x \rightarrow +\infty} \frac{2}{x} = 0$$

$$\text{vi) } \lim_{x \rightarrow -\infty} \frac{5x^3 - 2x^2 \eta \mu \frac{1}{x} + 2}{x^3 + x^2 \eta \mu \frac{2}{x} + 3}$$

$$\bullet \lim_{x \rightarrow -\infty} 2x^2 \eta \mu \frac{1}{x} = \lim_{u \rightarrow 0^-} 2 \frac{1}{u^2} \cdot \eta \mu u = \lim_{u \rightarrow 0^-} \frac{2}{u} \cdot \frac{\eta \mu u}{u} = -\infty \cdot 1 = -\infty$$

$$\text{Θ \acute{E} TW} \quad u = \frac{1}{x} < 0 \quad \lim_{x \rightarrow -\infty} u = 0$$

$$\lim_{x \rightarrow -\infty} \frac{\cancel{x^3} \left(5 - \frac{2x^2 \eta \mu \frac{1}{x}}{x^3} + \frac{2}{x^3} \right)}{\cancel{x^3} \left(1 + \frac{x^2 \eta \mu \frac{2}{x}}{x^3} + \frac{3}{x^3} \right)}$$

$$= \lim_{x \rightarrow -\infty} \frac{5 - \frac{2 \frac{1}{x} \eta \mu \frac{1}{x}}{x^3} + \frac{2}{x^3}}{1 + \frac{1}{x} \eta \mu \frac{2}{x} + \frac{3}{x^3}} \stackrel{u = \frac{1}{x}}{=} \lim_{u \rightarrow 0} \frac{5 - 2u \eta \mu u + 2u^3}{1 + u \eta \mu 2u + 3u^3}$$

$$= \frac{5-0+0}{1+0+0} = 5$$

2.17 Να υπολογίσετε τα παρακάτω όρια

$$\text{i) } \lim_{x \rightarrow -\infty} \frac{3x^2 - \sqrt{x^2 + \eta \mu^2 x}}{x^2 - \sqrt{x^2 + \sigma \nu \nu^2 x}}$$

$$\text{ii) } \lim_{x \rightarrow -\infty} \frac{5x^3 - 2x^2 \eta \mu \frac{1}{x} + 2}{x^3 - x^2 \eta \mu \frac{2}{x} + 3}$$