

2.15 Να υπολογίσετε τα παρακάτω όρια:

$$\alpha) \lim_{x \rightarrow -\infty} \frac{2}{x^2} \eta \mu \frac{1}{x}$$

$$\beta) \lim_{x \rightarrow +\infty} (x^2 + \eta \mu x) \sigma \upsilon \nu \frac{1}{x}$$

$$\alpha) \text{ Θέτω } u = \frac{1}{x} \Rightarrow x = \frac{1}{u}$$

$$x^2 = \frac{1}{u^2} \Rightarrow u^2 = \frac{1}{x^2}$$

$$\lim_{x \rightarrow -\infty} u = \lim_{x \rightarrow -\infty} \frac{1}{x} = 0$$

$$\lim_{u \rightarrow 0} 2u^2 \cdot \eta \mu u = 0$$

$$\beta) \lim_{x \rightarrow +\infty} (x^2 + \eta \mu x) \sigma \upsilon \nu \frac{1}{x} = (+\infty) \cdot (+\infty) = +\infty$$

$$= \lim_{x \rightarrow +\infty} \left(\frac{x^2 + \eta \mu x}{x} \right) \cdot x \sigma \upsilon \nu \frac{1}{x} = \lim_{x \rightarrow +\infty} \left(x + \frac{\eta \mu x}{x} \right) \cdot x \sigma \upsilon \nu \frac{1}{x}$$

$$\bullet \lim_{x \rightarrow +\infty} \frac{\eta \mu x}{x} = 0 \quad \alpha \rho \alpha \quad \lim_{x \rightarrow +\infty} \left(x + \frac{\eta \mu x}{x} \right) = +\infty$$

$$\bullet \lim_{x \rightarrow +\infty} x \sigma \upsilon \nu \frac{1}{x} = \lim_{u \rightarrow 0} \frac{1}{u} \cdot \sigma \upsilon \nu u = (+\infty) \cdot 1 = +\infty$$

$$\text{Θέτω } u = \frac{1}{x} \quad \lim_{x \rightarrow +\infty} u = 0$$

2.16 Να υπολογίσετε τα παρακάτω όρια:

$$i) \lim_{x \rightarrow +\infty} \frac{x \eta \mu^2 x}{x^4 + 1}$$

$$i) \lim_{x \rightarrow +\infty} \frac{\frac{x \eta \mu^2 x}{x^3}}{\frac{x^4 + 1}{x^3}} = \lim_{x \rightarrow +\infty} \frac{\frac{\eta \mu^2 x}{x^2}}{x + \frac{1}{x^3}}$$

$$\bullet \lim_{x \rightarrow +\infty} \frac{\eta \mu x}{x} = 0 \Rightarrow \lim_{x \rightarrow +\infty} \frac{\eta \mu^2 x}{x^2} = 0$$

$$\text{Έτσι } \lim_{x \rightarrow +\infty} \left(\frac{\eta \mu x}{x} \right)^2 \cdot \frac{1}{x + \frac{1}{x^3}} = 0 \cdot 0 = 0$$

$$E_{762} \quad \lim_{x \rightarrow +\infty} \left(\frac{\sqrt{x}}{x} \right)^2 \cdot \frac{1}{x + \frac{1}{x^3}} = 0 \cdot 0 = 0$$

$$v) \lim_{x \rightarrow +\infty} \left(\frac{x^3 + 2}{x^2 + 1} \cdot \eta \mu \frac{1}{x} \right)$$

$$\lim_{x \rightarrow +\infty} \frac{x^3 + 2}{x(x^2 + 1)} \cdot x \cdot \frac{1}{x} = \lim_{x \rightarrow +\infty} \frac{x^3 + 2}{x^3 + x} \cdot x \cdot \frac{1}{x} = 1 \cdot 1 = 1$$

$$\bullet \lim_{x \rightarrow +\infty} \frac{x^3 + 2}{x^3 + x} = \lim_{x \rightarrow +\infty} \frac{x^3}{x^3} = 1$$

$$\bullet \lim_{x \rightarrow +\infty} x \cdot \frac{1}{x} = \lim_{u \rightarrow 0} \frac{\eta \mu}{u} = 1$$

$\text{Öz. w. } u = \frac{1}{x} \quad \lim_{x \rightarrow +\infty} u = 0$