



'Ενα όριο για $x \rightarrow +\infty$

$$\lim_{x \rightarrow \infty} \frac{\ln(x^{4040} - 4x)}{\ln(x^2 - 2x)} =$$

$$\lim_{x \rightarrow \infty} \frac{\ln \left(x^{4040} \left(1 - \frac{4}{x^{4039}} \right) \right)}{\ln \left(x^2 \left(1 - \frac{2}{x} \right) \right)} =$$

$$\lim_{x \rightarrow \infty} \frac{\ln(x^{4040}) + \ln\left(1 - \frac{4}{x^{4039}}\right)}{\ln(x^2) + \ln\left(1 - \frac{2}{x}\right)}$$


$$\lim_{x \rightarrow \infty} \frac{4040 \cdot \ln x + \ln \left(1 - \frac{4}{x^{4039}} \right)}{2 \ln x + \ln \left(1 - \frac{2}{x} \right)} =$$

$$\lim_{x \rightarrow \infty} \frac{(\ln x) \left[4040 + \frac{\ln \left(1 - \frac{4}{x^{4039}} \right)}{\ln x} \right]}{(\ln x) \cdot \left[2 + \frac{\ln \left(1 - \frac{2}{x} \right)}{\ln x} \right]} =$$

$$\lim_{x \rightarrow \infty} \frac{\left[4040 + \frac{\ln \left(1 - \frac{4}{x^{4039}} \right)}{\ln x} \right]}{\left[2 + \frac{\ln \left(1 - \frac{2}{x} \right)}{\ln x} \right]} = \dots =$$

$$\frac{4040 + \frac{\ln(1+0)}{\ln(+\infty)}}{2 + \frac{\ln(1-0)}{\ln(+\infty)}} = \frac{4040 + \frac{0}{+\infty}}{2 + \frac{0}{+\infty}} = 2020$$