Ασκήσεις Λογάριθμοι.

Πέμπτη, 5 Μαΐου 2022 4:44 μμ

(4)

(v)

$$2\log_2(2+\sqrt{2}) + \log_2(6-4\sqrt{2}) =$$

$$\log_2(2+\sqrt{2})^2 + \log_2(6-4\sqrt{2}) =$$

$$\log_2\left[(2+\sqrt{2})^2 \cdot (6-4\sqrt{2})\right] =$$

$$\log_2\left[(4+4\sqrt{2}+2)(6-4\sqrt{2})\right] =$$

$$\log_2\left[(6+4\sqrt{2})(6-4\sqrt{2})\right] =$$

$$\log_2\left[6^2-(4\sqrt{2})^2\right]$$

$$\log_2(36-16\cdot 2)$$

$$\log_2(36-32)$$

$$\log_2 4 = 2.$$

(2) Να υπολογίσουν οι παρακάτω παραστάσεις.

α) $2\log_3 6 + \log_3 24 + \frac{1}{2}\log_3 4$

β) $\dfrac{\log_2 15 + \log_2 \frac{1}{6} - \log_2 10}{2\log_2 6 - \log_2 12}$

γ) $\dfrac{\frac{1}{2}\log_4 25 + \log_4 40 - \frac{3}{2}}{\log_4 10 - \frac{1}{2}}$

δ) $\dfrac{2\log_3 2 + \log_3 18 - 2}{\log_3 12 - 1}$

Λύση

α) $2\log_3 6 + \log_3 24 + \frac{1}{2}\log_3 4 =$

$\log_3 6^2 + \log_3 24 + \log_3 4^{1/2} =$

$\log_3 36 + \log_3 24 + \log_3 \sqrt{4} =$

$\log_3 36 + \log_3 24 + \log_3 2 =$

$\log_3 36 \cdot 24 \cdot 2 =$

$\log_3 1728$

β) $\dfrac{\log_2 15 + \log_2 \frac{1}{6} - \log_2 10}{2\log_2 6 - \log_2 12} =$

$\dfrac{\log_2 \frac{15 \cdot \frac{1}{6}}{10}}{\log_2 6^2 - \log_2 12} =$

$\dfrac{\log_2 \frac{\cancel{15}^{3}}{\cancel{6}^{2} \cdot \cancel{10}_{2}}}{\log_2 36 - \log_2 12} =$

$\dfrac{\log_2 \frac{1}{4}}{\log_2 \frac{36}{12}} =$

$\dfrac{\log_2 4^{-1}}{\log_2 3} =$

$$-\frac{\log_2 4}{\log_2 3} = \frac{-2}{\log_2 3}$$

δ)
$$\frac{\frac{1}{2}\log_4 25 + \log_4 40 - \frac{3}{2}}{\log_4 10 - \frac{1}{2}} =$$

$$\frac{\log_4 25^{1/2} + \log_4 40 - \frac{3}{2}\cdot 1}{\log_4 10 - \frac{1}{2}\cdot 1} =$$

$$\frac{\log_4 \sqrt{25} + \log_4 40 - \frac{3}{2}\log_4 4}{\log_4 10 - \frac{1}{2}\log_4 4} =$$

$$\frac{\log_4 5 + \log_4 40 - \log_4 4^{3/2}}{\log_4 10 - \log_4 4^{1/2}} =$$

$$\frac{\log_4 5 + \log_4 40 - \log_4 \sqrt{4^3}}{\log_4 10 - \log_4 \sqrt{4}} =$$

$$\frac{\log_4 5 + \log_4 40 - \log_4 2^3}{\log_4 10 - \log_4 2} =$$

$$\frac{\log_4 5 + \log_4 40 - \log_4 8}{\log_4 \frac{10}{2}} =$$

$$\frac{\log_4 \frac{5\cdot 40 5}{8}}{\log_4 5} =$$

$$\frac{\log_4 25}{} =$$

$$\frac{\log_4 5}{\log_4 5^2}$$

$$\frac{\log_4 5}{2\log_4 5} = \frac{1}{2}$$

δ)
$$\frac{2\log_3 2 + \log_3 18 - 2}{\log_3 12 - 1}$$

$$\frac{\log_3 2^2 + \log_3 18 - 2\cdot\log_3 3}{\log_3 12 - \log_3 3} =$$

$$\frac{\log_3 4 + \log_3 18 - \log_3 9}{\log_3 \frac{12}{3}} =$$

$$\frac{\log_3 \frac{4\cdot 18}{9}}{\log_3 4} =$$

$$\frac{\log_3 8}{\log_3 4} =$$

$$\frac{\log_3 2^3}{\log_3 2^2} =$$

$$\frac{3\log_3 2}{2\log_3 2} = \frac{3}{2}$$

(3)

Να βρεθούν οι τιμές των παραστάσεων:

$2\log 6 - \log 18$

Να βρεθούν οι τιμές των παραστάσεων:

α) $10^{2\log 6 - \log 12}$

β) $10^{3 - \log 5}$

γ) $10^{2 - \frac{1}{2}\log 16}$

δ) $\dfrac{10^{2 - \frac{1}{2}\log 5}}{5^{\log\sqrt{1.000}}}$

Λύση

α) $10^{2\log 6 - \log 12} =$

$10^{\log 6^2 - \log 12} =$

$10^{\log 36 - \log 12} =$

$10^{\log \frac{36}{12}} =$

$10^{\log 3} = 3$

β) $10^{3 - \log 5} =$

$10^{3\log 10 - \log 5} =$

$10^{\log 10^3 - \log 5} =$

$10^{\log \frac{10^3}{5}} = \frac{10^3}{5} = \frac{1000}{5} = 200$

γ) $10^{2 - \frac{1}{2}\log 16} =$

$10^{2\log 10 - \log 16^{1/2}}$

$10^{\log 10^2 - \log \sqrt{16}}$

$$\log_{10} 10^2 - \log_{10} \sqrt{16}$$

$$\log_{10} 100 - \log_{10} 4$$

$$\log_{10} \frac{100}{4}$$

$$\log_{10} 25 = 25$$

(δ)

$$\frac{10^{2 - \frac{1}{2}\log_5 5}}{\log_5 \sqrt{1000}} =$$

$$\frac{10^{2\log_{10} 10 - \log_5 5^{1/2}}}{\log_5 10 \cdot \sqrt{10}} =$$

$$\frac{10^{\log_{10} 100 - \log_5 \sqrt{5}}}{\log_5 10 + \log_5 \sqrt{10}} =$$

$$\frac{10^{\log_{10} \frac{100}{\sqrt{5}}}}{1 + \log_5 10^{1/2}} =$$

$$\frac{\frac{100 \cdot \sqrt{5}}{\sqrt{5}\sqrt{5}}}{5^1 \cdot 5^{\frac{1}{2}\log_5 10}} =$$

$$\frac{20\,\frac{100}{5}\sqrt{5}}{5 \cdot 5^{1/2}} =$$

$$\frac{20\sqrt{5}}{5\sqrt{5}} =$$

$$\frac{20}{5} = 4$$

$$\frac{20}{5} = 4$$

④

Εάν $\log_\kappa 2 = a$ και $\log_\kappa 3 = \beta$ με $0 < \kappa \neq 1$, να δειχθεί ότι:

$$\log_\kappa \left(\frac{9}{2} \cdot \kappa^5\right) = 2\beta - a + 5$$

Λύση

$$\log_\kappa \left(\frac{9}{2} \cdot \kappa^5\right) =$$

$$\log_\kappa \frac{9}{2} + \log_\kappa \kappa^5 =$$

$$\log_\kappa 9 - \log_\kappa 2 + 5\log_\kappa \kappa$$

$$\log_\kappa 3^2 - \log_\kappa 2 + 5\log_\kappa \kappa$$

$$2\log_\kappa 3 - \log_\kappa 2 + 5\log_\kappa \kappa^1$$

$$2\beta - a + 5$$