CHAPTER 1.4
WORKSHEET: MULTIPLICATION OF POLYNOMIALS

1. Theory Summary – Definitions

Polynomial multiplication is based on the distributive property: $\alpha\cdot(\beta+\gamma)=\alpha\beta+\alpha\gamma$ *and the double distributive property:* $(\alpha+\beta)\cdot(\gamma+\delta)=\alpha\gamma+\alpha\delta+\beta\gamma+\beta\delta$

Multiplication of a Monomial by a Polynomial

To multiply a monomial by a polynomial, we multiply the monomial by each term of the polynomial and add the resulting products.

Example: $4x^2\cdot(3x^2-2x+4)=4x^2\cdot(3x^2)+4x^2\cdot(-2x)+4x^2\cdot 4=12x^4-8x^3+16x^2$

Multiplication of a Polynomial by a Polynomial

To multiply a polynomial by another polynomial, we multiply each term of one polynomial by each term of the other polynomial and add the resulting products.

Example:

$(2x+3)(-x^2+2x+4)=2x\cdot(-x^2)+2x\cdot(2x)+2x\cdot 4+3\cdot(-x^2)+3\cdot(2x)+3\cdot 4=$

$=-2x^3+4x^2+8x-3x^2+6x+12=-2x^3+4x^2-3x^2+8x+6x+12=-2x^3+x^2+14x+12$

The degree of the product of polynomials is equal to the sum of the degrees of the original polynomials.

Example: If $P(x)=3x^4+4x^2-19x$ is of 4th degree and $Q(x)=x^2-4x+1$ is of 2nd degree, then their product $P(x)\cdot Q(x)$ is of 6th degree.

A root of a polynomial is any number that makes the polynomial equal to zero.

Example: If $P(x)=x^2-2x+1$ and we set x = 1, then $P(1)=1^2-2\cdot 1+1=1-2+1=0$. Therefore, the number 1 is a root of the polynomial P(x).

Exercises

B1. Find the expansions (products):

α) $(3x^2-2x)(4x^3+4x)$

β) $(x^4-2x+2)(x^5-2x)$

γ) $(x-3)(x-2)x$

δ) $(\alpha^2-3\alpha+1)(\alpha-2)+(\alpha^3-2\alpha)(\alpha^2-\alpha)$

B2. If $P(x)=2x(-x^2+3x)(x-2)$ and $Q(x)=\alpha(x^4+1)+\beta x^3+\gamma x^2+1$, find the values of α, β, γ so that the polynomials P(x) and Q(x) are equal.

B3. Given the polynomial $P(x) = 2x^2$:

a) Determine its degree.

b) Find the polynomial $Q(x) = P(P(x)) - 1$.

B4. To produce x units of a product per week, a company incurs cost $K(x) = 500x + 50000$ euros. The company sells these x units per week at a price of $T(x) = 2000 - 2x$ euros per unit.

a) Find the polynomial representing the profit from selling x units of the product per week.

b) Find the cost when producing 0 units and justify your answer.

c) Calculate the profit if the company sells 100 units per week.