CHAPTER 1.1 Γ: WORKSHEET — SQUARE ROOT OF REAL NUMBERS

1. Theory Summary

Definition of Square Root: The square root of a positive number x is defined as the positive number $a=\sqrt{x}$ for which: if the number a is squared, it gives the number x. That is: $x = a^2$

ATTENTION! The square root of a negative number does not exist in real numbers.

Since $a=\sqrt{x}$, it holds that $a^2 = x$ and $\sqrt{(x^2)}=|x|$. (In the case where $x\geq 0$, $\sqrt{(x^2)}=x$).

Property	Mathematical Formula	Example / Note
Product of Square Roots	$\sqrt{(a\cdot b)}=sqrta\cdot sqrtb$	$\sqrt{5}\cdot 6=\sqrt{5}\cdot\sqrt{6}$
Quotient of Square Roots	$\sqrt{\frac{\alpha}{\beta}}=\frac{\sqrt{\alpha}}{\sqrt{\beta}}$, $\beta\neq 0$	$\sqrt{\frac{2}{x}}=\frac{\sqrt{2}}{\sqrt{x}}$, $x\neq 0$
CRITICAL WARNING (Addition & Subtraction)	$\sqrt{a+\beta}\neq\sqrt{a}+\sqrt{\beta}$ $\sqrt{a-\beta}\neq\sqrt{a}-\sqrt{\beta}$, $\alpha,\beta\geq 0$	Valid for positive numbers a, b, x (a ≥ 0, b > 0, x > 0)

FRACTIONS WITH A RADICAL IN THE DENOMINATOR:

The expressions $\sqrt{a}+\sqrt{\beta}$ and $\sqrt{a}-\sqrt{\beta}$ are called ***conjugate expressions***. In algebra, we prefer fractions not to contain square roots in the denominator. To eliminate the root from the denominator, we apply the following methodology:

Methodology: If the denominator contains only a radical or a product of radicals, we multiply both the numerator and the denominator by the radical(s).

Example: $\frac{3}{\sqrt{5}\cdot\sqrt{3}}=\frac{3\cdot\sqrt{5}\cdot\sqrt{3}}{\sqrt{5}\cdot\sqrt{3}\sqrt{5}\cdot\sqrt{3}}=\frac{3\cdot\sqrt{5}\cdot\sqrt{3}}{5\cdot 3}=\frac{3\cdot\sqrt{5}\cdot\sqrt{3}}{15}$

TOPIC A: Theory & Comprehension Questions

A1. Characterize the following statements as True (T) or False (F):

a) It holds that $\sqrt{a^2}=a$ for every real number a. [ ]

b) It holds that $\sqrt{(-4)^2}=-4$. [ ]

c) It holds that $\sqrt{a^2\cdot\beta^2}=\sqrt{a^2}\cdot\sqrt{\beta^2}$ for any real numbers a, b. [ ]

d) It holds that $\sqrt{a^2+\beta}=|a|+\sqrt{\beta}$ for every real a and non-negative b. [ ]

e) It holds that $\sqrt{4}=2$ or $\sqrt{4}=-2$. [ ]

A2. Complete the following table:

a	b	$\sqrt{a}$	$\sqrt{\beta}$	$\sqrt{a+\beta}$	$\sqrt{a}+\sqrt{\beta}$	$\sqrt{a\cdot\beta}$	$\sqrt{a}\cdot\sqrt{\beta}$
4	49						
25	324						
169	196						

TOPIC B: Application of Root Properties

B1. Simplify the following expressions:

α) $\sqrt{2}\cdot\sqrt{50}+\sqrt{7+\sqrt{4}}-\sqrt{3-\sqrt{4}}$

β) $5\sqrt{\sqrt{16}}-\sqrt{20\cdot\sqrt{25}}+\sqrt{2\cdot\sqrt{64}}$

γ) $\sqrt{\frac{2}{3}}\cdot\sqrt{\frac{6}{2}}\cdot\sqrt{\frac{8}{4}}$

δ) $\sqrt{(3-\sqrt{5})^2}+\sqrt{(1-\sqrt{5})^2}$

ε) $\sqrt{13+\sqrt{7+\sqrt{3+\sqrt{1}}}}$

στ) $\sqrt{72}-2\sqrt{50}+4\sqrt{162}-10\sqrt{32}$

B2. Convert the following fractions into equivalent ones with a rational denominator:

α) $\frac{1}{\sqrt{2}}$ β) $\frac{3}{2\cdot\sqrt{3}}$ γ) $\frac{4}{-\sqrt{5}}$ δ) $\frac{\sqrt{2}-1}{\sqrt{3}}$