

CHAPTER 1.1 B

WORKSHEET: POWERS OF REAL NUMBERS

1. Summary of Theory & Properties of Powers

Property	Mathematical Formula	Example / Note
Product of powers with the same base	$a^\mu \cdot a^\nu = a^{(\mu+\nu)}$	$2^3 \cdot 2^4 = 2^7 = 128$
Quotient of powers with the same base	$\frac{a^\mu}{a^\nu} = a^{(\mu-\nu)}$	$\frac{5^6}{5^4} = 5^2 = 25$
Power of a product	$(a \cdot \beta)^\nu = a^\nu \cdot \beta^\nu$	$(2 \cdot x)^3 = 8x^3$
Power of a quotient	$\left(\frac{a}{\beta}\right)^\nu = \frac{a^\nu}{\beta^\nu}$	$\left(\frac{3}{4}\right)^2 = \frac{9}{16}$
Power of a power and Negative Exponent	$(a^\mu)^\nu = a^{(\mu \cdot \nu)}$ $a^{(-\nu)} = \frac{1}{a^\nu}$	$(3^2)^3 = 3^6$ $2^{(-3)} = \frac{1}{8}$

TOPIC A: Theory & Comprehension Questions

A1. Classify the following statements as True (T) or False (F):

- a) For every non-zero real number a , $a^0 = 1$ holds true. []
- b) The power $(-3)^4$ is a negative number because the base is negative. []
- c) $(a + \beta)^2 = a^2 + \beta^2$ holds true for any numbers a, β . []
- d) It holds true that $a^\mu \cdot a^\nu = a^{(\mu \cdot \nu)}$ for every $a \neq 0$. []
- e) The reciprocal of the number 2^{-3} is the number 2^3 . []

A2. Fill in the blanks to create true equalities:

- a) If $a \neq 0$ and n is a natural number, then $a^{-n} = \dots\dots\dots$
- b) If the base a is a negative number and the exponent n is even, then the sign of the power a^n is $\dots\dots\dots$
- c) The product $(a \cdot \beta)^\nu$ is equal to $\dots\dots\dots$

TOPIC B: Application of Power Properties

B1. Write the following expressions as a single power:

a) $3^4 \cdot 3^5 \cdot 3^{-2}$

β) $\frac{(2^3)^4}{2^5}$

γ) $\frac{(-5)^7}{(-5)^3}$

$$\delta) \frac{(-5)^7}{(-5)^3}$$

B2. Calculate the value of the numerical expressions:

$$\alpha) A = (-1)^{2024} + (-1)^{2025} - (-2)^3$$

$$\beta) B = 2^{-3} + 4^{-1} - \left(\frac{1}{2}\right)^{-2}$$

$$\gamma) \Gamma = \left(\frac{1}{3}\right)^{-2} \cdot 3^{-3} + \frac{(-2)^4}{2^3}$$

TOPIC C: Complex Numerical Expressions

$$\mathbf{C1.} \text{ Given the expressions: } A = \frac{[(-2)^3 \cdot 2^{-4}]^{-2}}{4^2} \text{ and } B = \frac{3^5 \cdot 9^{-2}}{27^{-1}} \cdot 3^4$$

a) Simplify expressions A and B, showing that $A = 1$ and $B = 3$.

b) Calculate the value of the expression $K = A^{-5} + B^2 - (A + B)^0$.

$$\mathbf{C2.} \text{ Simplify the complex fraction : } \Delta = \frac{2^{-2} + \left(\frac{1}{2}\right)^{-1}}{3^{-1} - \left(\frac{1}{3}\right)^2}$$

$$\mathbf{TOPIC D: D1.} \text{ Given the algebraic expression : } P(x, y) = \frac{(x^{-2} \cdot y^3)^{-2} \cdot (x^3 \cdot y^{-1})^4}{(x^8 \cdot y^{-2})}$$

a) Simplify the expression $P(x, y)$ so that it takes the form $x^a \cdot y^b$.

b) Calculate the value of the expression $P(x, y)$ for $x = 2026$ and .

$$\mathbf{D2.} \text{ If } n \text{ is a natural number, prove that the value of the expression : } \Lambda = \frac{3^{n+2} - 3^n}{3^{n+1} + 3^n} \text{ is}$$

independent of n and find its value.