

$$7) a) \left. \begin{array}{l} \varepsilon_1: y = \sqrt{3}x + 6 \quad : \lambda_1 = \sqrt{3} \\ \varepsilon_2: y = \sqrt{3}x - 2 \quad : \lambda_2 = \sqrt{3} \end{array} \right\} \Rightarrow \lambda_1 = \lambda_2$$

$\Downarrow$
 $\varepsilon_1 \parallel \varepsilon_2$

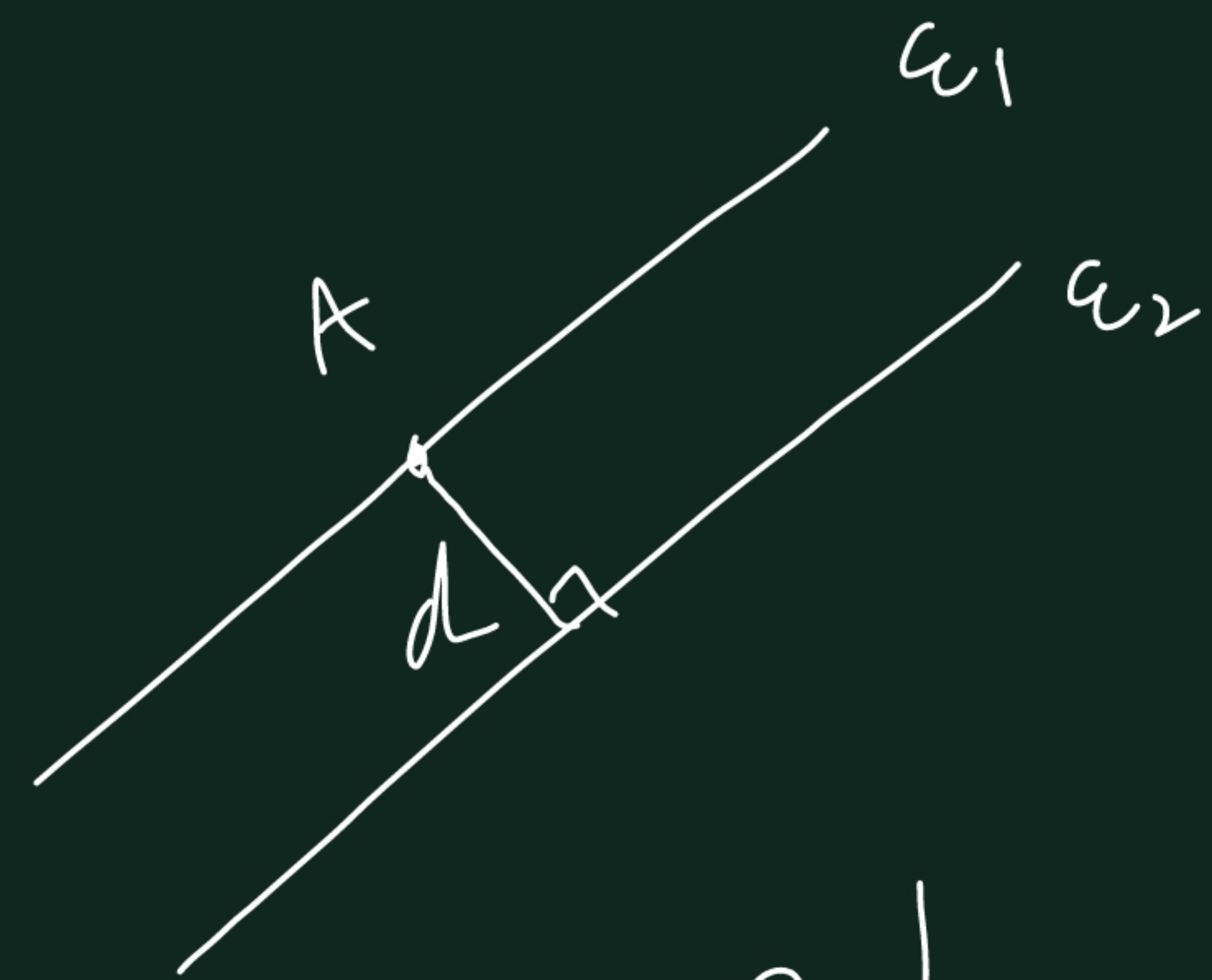
$$\varepsilon: y = \lambda x + b$$

b) $d(\varepsilon_1, \varepsilon_2) = ?$

$\hookrightarrow \boxed{\varepsilon_2: \sqrt{3}x - y - 2 = 0}$

$\varepsilon_1 \xrightarrow{x=0} y=6 \Rightarrow A(0, 6) \in \varepsilon_1$

Apu $d(\varepsilon_1, \varepsilon_2) = d(A, \varepsilon_2) = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} = \frac{|\sqrt{3} \cdot 0 - 1 \cdot 6 - 2|}{\sqrt{\sqrt{3}^2 + (-1)^2}} =$



$$\frac{|-6-2|}{\sqrt{4}} = \frac{|-8|}{2} = \frac{8}{2} = 4 \Rightarrow \boxed{d(\varepsilon_1, \varepsilon_2) = 4}$$

$$9) \quad \varepsilon_1: 4x - 3y + 24 = 0$$

$$\varepsilon_2: 4x + 3y = 0$$

$$d(M, \varepsilon_1) = d(M, \varepsilon_2)$$

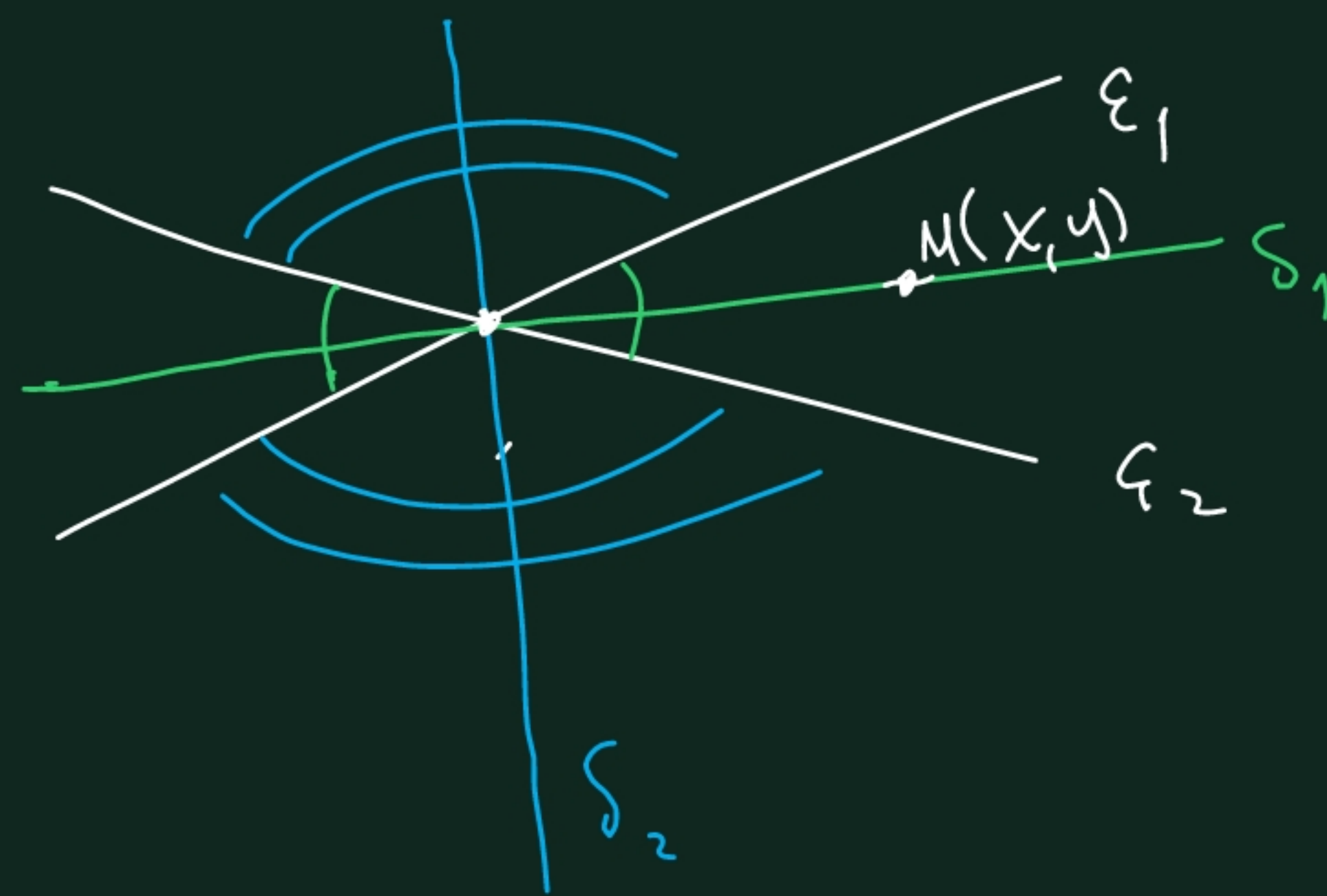
$$\Leftrightarrow \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}} = \frac{|Ax_0 + By_0 + C|}{\sqrt{A^2 + B^2}}$$

$$\Leftrightarrow \frac{|4x - 3y + 24|}{\sqrt{4^2 + 3^2}} = \frac{|4x + 3y|}{\sqrt{4^2 + 3^2}}$$

$$\Leftrightarrow \frac{|4x - 3y + 24|}{\sqrt{4^2 + 3^2}} = \frac{|4x + 3y|}{\sqrt{4^2 + 3^2}}$$

$$\Leftrightarrow |4x - 3y + 24| = |4x + 3y|$$

Να βρεθούν οι εφιαύρες των
σταθερών των ευθειών των $\varepsilon_1, \varepsilon_2$



$$|a| = |b| \Leftrightarrow$$

$$a = b \text{ ή } a = -b$$

$$1) 4x - 3y + 24 = 4x + 3y \Rightarrow -6y = -24 \Rightarrow y = 4$$

$$2) 4x - 3y + 24 = -4x - 3y \Rightarrow$$

$$4x + 4x + 3y + 3y = -24 \Rightarrow \frac{8x}{8} = \frac{-24}{8}$$

$$x = -3$$

$$8) \quad \left. \begin{array}{l} \varepsilon_1: 6x + 8y + 1 = 0 \\ \varepsilon_2: 12x - 5y - 3 = 0 \end{array} \right\} \quad \text{---}$$

$$\text{Eow } M(x, y) \in \delta_1$$

$$d(M, \varepsilon_1) = d(M, \varepsilon_2) \Leftrightarrow$$

$$\frac{|6x + 8y + 1|}{\sqrt{36 + 64}} = \frac{|12x - 5y - 3|}{\sqrt{144 + 25}} \quad \Leftrightarrow$$

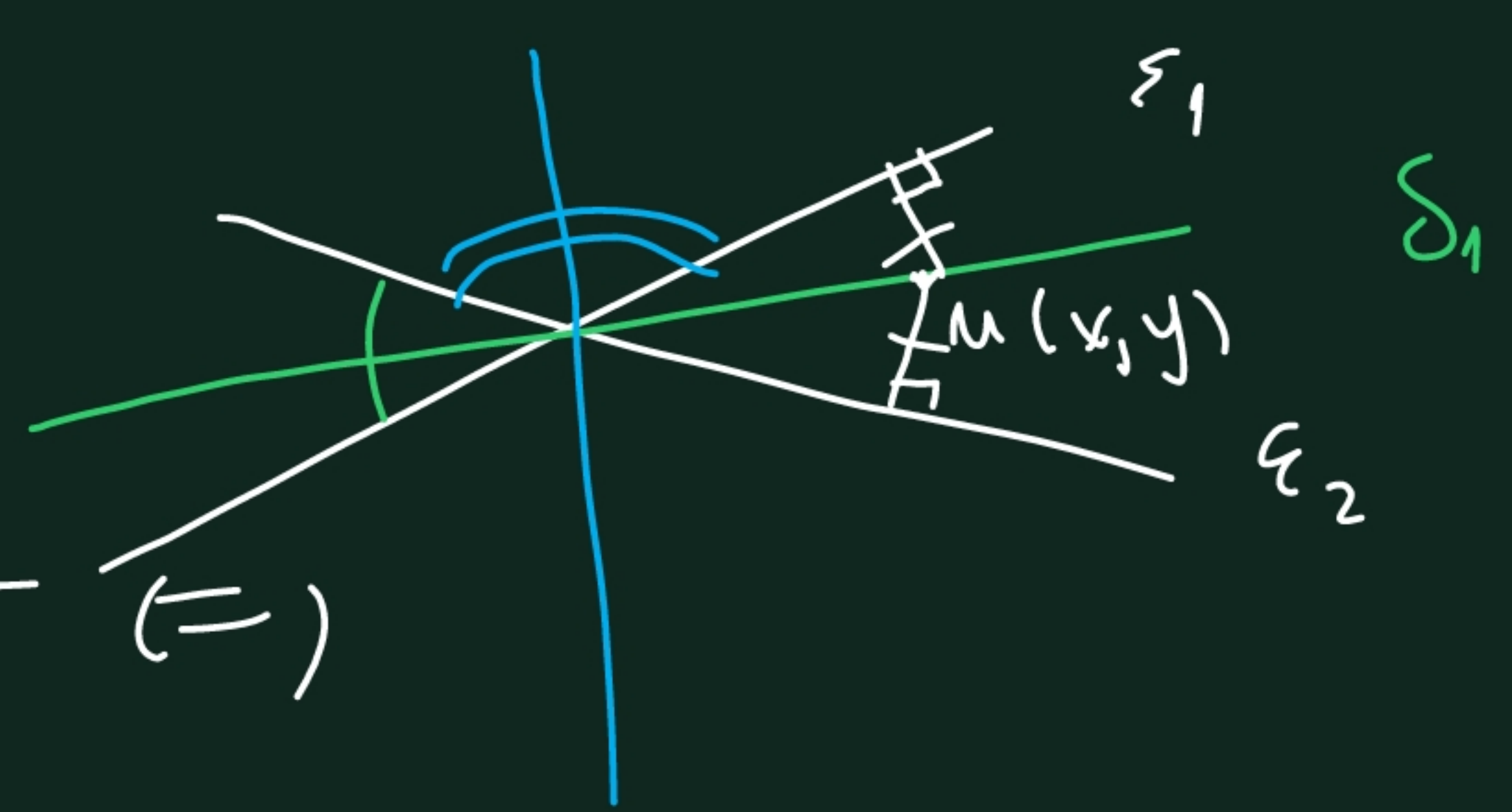
$$\left\{ \begin{array}{l} 78x + 104y + 13 = 120x - 50y - 30 \\ 78x + 104y + 13 = -120x + 50y + 30 \end{array} \right\} \quad \Leftrightarrow$$

$$\frac{|6x + 8y + 1|}{10} = \frac{|12x - 5y - 3|}{13} \quad \Leftrightarrow$$

$$13|6x + 8y + 1| = 10|12x - 5y - 3| \quad \Leftrightarrow$$

$$1) \quad 13(6x + 8y + 1) = 10(12x - 5y - 3) \quad \Leftrightarrow$$

$$2) \quad 13(6x + 8y + 1) = -10(12x - 5y - 3)$$



$$\left\{ \begin{array}{l} \delta_1: -72x + 154y + 43 = 0 \\ \delta_2: 198x + 54y - 17 = 0 \end{array} \right.$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, \quad b = \sqrt{a^2 - b^2}$$

$$E', E \in x'x$$

$$E(b, 0), E'(-b, 0)$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1, \quad b = \sqrt{a^2 - b^2}$$

$$E(0, b), E'(0, -b)$$

i) $E'(-4, 0), E(4, 0) \rightarrow \textcircled{y=4} : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$\rightarrow 2a = 10 \Rightarrow \textcircled{a=5}$

$$E_{\text{low}} \quad b = \sqrt{25 - 16} = \sqrt{9} = 3 \quad \textcircled{b=3}$$

$$\text{Apu } C: \frac{x^2}{25} + \frac{y^2}{9} = 1$$

ii) $E'(0, -5)$ $2a = 26 \Rightarrow \textcircled{a=13}$

$E(0, 5) \rightarrow y' \text{ μέση } A \text{ foron}$

$\rightarrow f=5$

$$b = \sqrt{169 - 25} = \sqrt{144} = \textcircled{12}$$

$$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \Rightarrow C: \frac{x^2}{144} + \frac{y^2}{169} = 1$$

$$2) i) C: x^2 + 4y^2 = 4 \quad (\stackrel{4}{=}) C: \frac{x^2}{4} + \frac{y^2}{1} = 1$$

$$AA': x \dot{x}$$

$$\left. \begin{array}{l} a^2 = 4 \Rightarrow a = 2 \\ b^2 = 1 \Rightarrow b = 1 \end{array} \right\}$$

Exw $b = \sqrt{a^2 - 1} (=1)$

$$1 = \sqrt{4 - b^2} \stackrel{\text{u. w. w.}^2}{=} 1$$

$$1 = 4 - b^2 \Rightarrow b^2 = 3 \Rightarrow b = \pm\sqrt{3}$$

$$AA': 2a = 2 \cdot 2 = \textcircled{4}$$

$$BB': 2b = 2 \cdot 1 = \textcircled{2}$$

$$\left| \begin{array}{l} E(\sqrt{3}, 0) \\ E'(-\sqrt{3}, 0) \end{array} \right|$$

$$e = \frac{b}{a} = \frac{\sqrt{3}}{2}$$

$$2) \text{ ii) } 169x^2 + 144y^2 = 24336 \quad (\Rightarrow \div 24 \cdot 336)$$

$$C: \frac{x^2}{144} + \frac{y^2}{169} = 1 \quad \leadsto \quad yy': AA'$$

$$a^2 = 169 \Rightarrow a = 13$$

$$b^2 = 144 \Rightarrow b = 12$$

$$exw \quad b = \sqrt{a^2 - f^2} \quad (=)$$

$$12 = \sqrt{169 - f^2} \quad (\stackrel{u_{f,w,v}}{=})$$

$$144 = 169 - f^2 \quad (=) \quad f^2 = 25 \Rightarrow f = \pm 5$$

$$AA': 2a = 26$$

$$BB': 2b = 24$$

$$E(0, 5)$$

$$E'(0, -5)$$

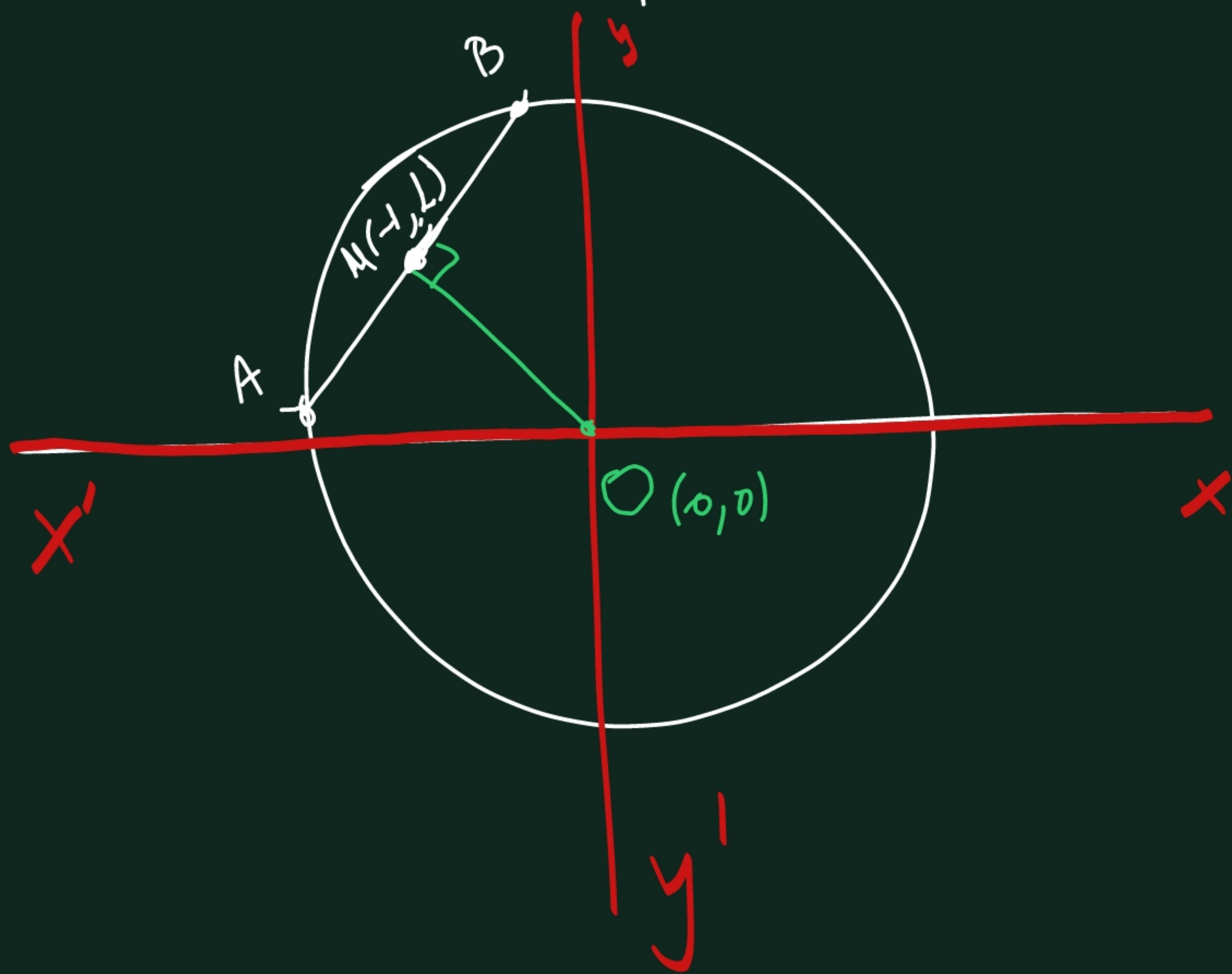
$$e = \frac{f}{a} = \frac{5}{13} \Rightarrow e = \frac{5}{13}$$

4)

$$C: x^2 + y^2 = 4 \Rightarrow \rho =$$

5. 87

Να βρεθεί η χορδή που έχει μέσο $M(-1, 1)$



OM : απόσταση

$$OM \perp AB \Leftrightarrow$$

$$\lambda_1 \cdot \lambda_2 = -1 \Leftrightarrow$$

$$\frac{y_2 - y_1}{x_2 - x_1} \cdot \lambda_1 = -1 \Leftrightarrow$$

$$\frac{1 - 0}{-1 - 0} \cdot \lambda_2 = -1 \Leftrightarrow$$

$$-1 \cdot \lambda_2 = -1 \Leftrightarrow \lambda_2 = 1$$

$$\underline{AB}: y - y_0 = \lambda(x - x_0) \xrightarrow[\lambda(-1,1)]{\lambda=1}$$

$$y - 1 = 1(x + 1) \Leftrightarrow$$

$$\boxed{AB: y = x + 2}$$

