• Αν $\vec{a} \perp \vec{b} \iff \lambda_1 \cdot \lambda_2 = -1$

$\vec{a}=(x,y) \quad \lambda = \frac{y}{x}$

Αποδ: Έστω $\vec{a}=(x_1,y_1)$, $\vec{b}=(x_2,y_2)$

Έχω $\vec{a} \perp \vec{b} \iff \vec{a}\cdot\vec{b} = 0 \iff$

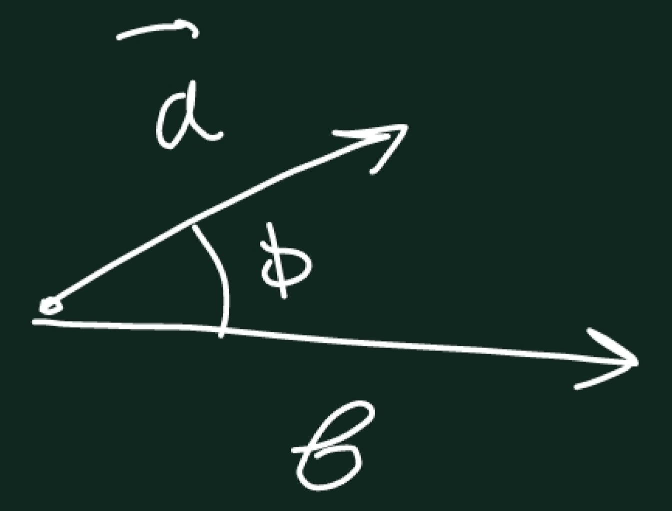
$$(x_1,y_1)\cdot(x_2,y_2) = 0 \iff x_1 \cdot x_2 + y_1 \cdot y_2 = 0 \iff$$

$$y_1 \cdot y_2 = -x_1 \cdot x_2 \underset{x_1 \neq 0 \text{ κ } x_2 \neq 0}{\overset{: x_1 \cdot x_2 \neq 0}{\iff}} \frac{y_1 \cdot y_2}{x_1 \cdot x_2} = -1 \iff$$

$$\frac{y_1}{x_1} \cdot \frac{y_2}{x_2} = -1 \iff \boxed{\lambda_1 \cdot \lambda_2 = -1}$$

Συνημίτονο γωνίας 2 διανυσμάτων

$\vec{a}=(x_1,y_1)$, $\vec{\beta}=(x_2,y_2)$



Έχω $\vec{a}\cdot\vec{\beta}=|\vec{a}|\cdot|\vec{\beta}|\cdot\text{συν}\hat{\phi} \overset{:|\vec{a}|\cdot|\vec{\beta}|}{\iff}$

$$\frac{\vec{a}\cdot\vec{\beta}}{|\vec{a}|\cdot|\vec{\beta}|}=\frac{\cancel{|\vec{a}|}\cdot\cancel{|\vec{\beta}|}\cdot\text{συν}\hat{\phi}}{\cancel{|\vec{a}|}\cdot\cancel{|\vec{\beta}|}} \iff \boxed{\text{συν}\hat{\phi}=\frac{\vec{a}\cdot\vec{\beta}}{|\vec{a}|\cdot|\vec{\beta}|}} \rightsquigarrow \boxed{\text{συν}\hat{\phi}=\frac{x_1\cdot x_2+y_1\cdot y_2}{\sqrt{x_1^2+y_1^2}\cdot\sqrt{x_2^2+y_2^2}}}$$

1) i) $\vec{a}=(-1,3)$, $\vec{\beta}=(2,5)$

σ.47

Να βρεθει το $(\vec{a}-\vec{\beta})\cdot(3\vec{a}+\vec{\beta}) = \Big((-1,3)-(2,5)\Big)\cdot\Big(3(-1,3)+(2,5)\Big) =$

$$(-3,-2)\Big((-3,9)+(2,5)\Big) = (-3,-2)\cdot(-1,14) = (-3)\cdot(-1)+(-2)\cdot 14 =$$

$$3-28 = -25$$

ii) Να βρεθεί η σχέση των $\kappa,\lambda \in \mathbb{R}$ τ.ω $\vec{u}=(\kappa,\lambda)$ και $\vec{\beta}=(2,5)$

να είναι $\vec{u}\cdot\vec{\beta}=0 \iff (\kappa,\lambda)\cdot(2,5)=0 \iff \boxed{2\cdot\kappa+5\cdot\lambda=0} \iff$

2) $\vec{u}=(1,2)$, $\vec{v}=(4,2)$, $\vec{w}=(6,0)$

σ. 47

$$|\vec{u}|\cdot\vec{v}\cdot\vec{w}=\sqrt{1^2+2^2}\cdot(4,2)\cdot(6,0)=\sqrt{5}\cdot(4\cdot 6+2\cdot 0)=\sqrt{5}\cdot 24$$

3) $\vec{a}=(1,0)$, $\vec{b}=(1,1)$ να βρεθεί $\lambda\in\mathbb{R}$

ii) $\vec{b}\perp(\vec{a}+\lambda\vec{b})\Leftrightarrow\vec{b}\cdot(\vec{a}+\lambda\vec{b})=0\Leftrightarrow\vec{b}\cdot\vec{a}+\lambda\vec{b}^2=0\Leftrightarrow$

$$(1,0)\cdot(1,1)+\lambda\cdot|\vec{b}|^2=0\Leftrightarrow 1\cdot 1+\cancel{1\cdot 0}^{\,0}+\lambda\cdot\cancel{\sqrt{1^2+1^2}}^{\,\cancel{2}}=0$$

$$1+\lambda\cdot 2=0\Leftrightarrow \frac{\cancel{2}\lambda}{\cancel{2}}=\frac{-1}{2}\overset{:2}{\Leftrightarrow}\boxed{\lambda=-1/2}$$

4) $\vec{u}=(3,-2)$, να βρεθεί $\vec{v}$ τ.ω $\vec{v}\perp\vec{u}$ και $|\vec{v}|=1$

σ. 47

Έστω $\vec{v}=(x,y)$

- $\vec{v}\perp\vec{u} \iff \vec{v}\cdot\vec{u}=0 \iff (x,y)\cdot(3,-2)=0 \iff x\cdot 3+y\cdot(-2)=0 \iff \boxed{3x-2y=0}\ (1)$

- $|\vec{v}|=1 \iff \sqrt{x^2+y^2}=1 \overset{\text{όλα θετ}}{\underset{\text{υψώνω}}{\iff}} \sqrt{x^2+y^2}^{\,2}=1^2 \iff \boxed{x^2+y^2=1}\ (2)$

$(1) \iff \dfrac{\cancel{3}x}{\cancel{3}}=\dfrac{2y}{3} \overset{:3}{\iff} \boxed{x=\frac{2}{3}\cdot y}\ (3) \quad (2) \longrightarrow \left(\frac{2}{3}y\right)^2+y^2=1 \iff$

$\frac{4}{9}y^2+y^2=1 \overset{\cdot 9}{\iff} 4y^2+9\cdot y^2=9 \iff \dfrac{\cancel{13}\cdot y^2}{\cancel{13}}=\dfrac{9}{13} \overset{:13}{\iff} y^2=\frac{9}{13} \iff y=\pm\frac{3}{\sqrt{13}} \Rightarrow \begin{cases} y_1=\frac{3\sqrt{13}}{13} \\ y_2=\frac{-3\sqrt{13}}{13}\end{cases}$

$(3) \Longrightarrow \begin{cases} x_1=\frac{2}{\cancel{3}}\cdot\frac{\cancel{3}}{\sqrt{13}} \Rightarrow \boxed{x_1=\frac{2}{\sqrt{13}}} \\ x_2=\frac{2}{\cancel{3}}\left(-\frac{\cancel{3}}{\sqrt{13}}\right) \Rightarrow \boxed{x_2=-\frac{2}{\sqrt{13}}} \end{cases}$

Άρα $\begin{cases} \vec{v}_1=\left(\frac{2}{\sqrt{13}},\frac{3}{\sqrt{13}}\right) \\ \vec{v}_2=\left(-\frac{2}{\sqrt{13}},-\frac{3}{\sqrt{13}}\right)\end{cases}$