14. β) $2\cdot\eta\mu^2x - 3\cdot\sigma\upsilon\nu(\pi+x) = 0 \iff$

$2\cdot\eta\mu^2x - 3\cdot(-\sigma\upsilon\nu x) = 0 \iff$

$2\cdot\eta\mu^2x + 3\cdot\sigma\upsilon\nu x = 0 \iff$

$2\cdot(1-\sigma\upsilon\nu^2x) + 3\cdot\sigma\upsilon\nu x = 0 \iff$

$2 - 2\cdot\sigma\upsilon\nu^2x + 3\cdot\sigma\upsilon\nu x = 0 \iff$

$-2\cdot\sigma\upsilon\nu^2x + 3\cdot\sigma\upsilon\nu x + 2 = 0$

Θέτω $\sigma\upsilon\nu x = w$, $-1 \le w \le 1$

$\eta\mu^2x + \sigma\upsilon\nu^2x = 1 \iff$

$\eta\mu^2x = 1 - \sigma\upsilon\nu^2x$

$-2w^2 + 3w + 2 = 0$

$\Delta = \beta^2 - 4\alpha\gamma = 3^2 - 4(-2)\cdot 2 =$

$9 + 16 = 25 > 0$

$w_{1,2} = \frac{-\beta \pm \sqrt{\Delta}}{2\alpha} = \frac{-3 \pm 5}{-4}$ $\overset{+}{\rightarrow} \frac{2}{-4} = -\frac{1}{2}$

$\rightarrow \frac{-8}{-4} = 2$ απορρ

Άρα $w = -\frac{1}{2} \iff \sigma\upsilon\nu x = -\frac{1}{2} \iff$

$\sigma\upsilon\nu x = -\sigma\upsilon\nu\frac{\pi}{3} \overset{2^{\alpha}}{\iff} \sigma\upsilon\nu x = \sigma\upsilon\nu\left(\pi - \frac{\pi}{3}\right) \iff$

$\sigma\upsilon\nu x = \sigma\upsilon\nu\frac{2\pi}{3} \iff$

$$x = 2k\pi \pm \frac{2\pi}{3},\ k \in \mathbb{Z}$$

15) α) $\eta\mu x = \sigma\upsilon\nu x \Leftrightarrow \eta\mu x = \eta\mu\left(\frac{\pi}{2} - x\right)$

$\eta\mu x = \eta\mu\theta \Leftrightarrow$

$x = 2k\pi + \theta$

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$x = 2k\pi + \pi - \theta$

$x = 2k\pi + \frac{\pi}{2} - x$ ①

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$x = 2k\pi + \pi - \left(\frac{\pi}{2} - x\right)$ ②

$k \in \mathbb{Z}$

① $\Leftrightarrow 2x = 2k\pi + \frac{\pi}{2} \overset{:2}{\Leftrightarrow}$, $\boxed{x = k\pi + \frac{\pi}{4}, k \in \mathbb{Z}}$

② $\Leftrightarrow$ ~~$x = 2k\pi + \pi - \frac{\pi}{2} + x$~~ $\Leftrightarrow 0 = 2k\pi - \frac{\pi}{2}$ Αδύνατη

15) β) $\eta\mu x = -\sqrt{3}\cdot \sigma\upsilon x \quad (:\sigma\upsilon x \neq 0)$ $\sigma\upsilon x \neq 0 \Leftrightarrow x \neq \kappa\pi + \frac{\pi}{2}$

$$\frac{\eta\mu x}{\sigma\upsilon x} = -\sqrt{3} \Leftrightarrow \varepsilon\varphi x = -\sqrt{3} \Leftrightarrow \varepsilon\varphi x = -\varepsilon\varphi\frac{\pi}{3} \overset{2\pi}{\Leftrightarrow}$$

$$\varepsilon\varphi x = \varepsilon\varphi\left(\pi - \frac{\pi}{3}\right) \Leftrightarrow \varepsilon\varphi x = \varepsilon\varphi\frac{2\pi}{3} \Leftrightarrow \boxed{x = \kappa\pi + \frac{2\pi}{3},\ \kappa \in \mathbb{Z}}$$

17)α) Να λυθεί $\eta\mu\left(2x-\frac{\pi}{3}\right)-\sigma\upsilon\nu 2x=0 \Leftrightarrow$

$\eta\mu\left(2x-\frac{\pi}{3}\right)=\sigma\upsilon\nu 2x \Leftrightarrow \eta\mu\left(2x-\frac{\pi}{3}\right)=\eta\mu\left(\frac{\pi}{2}-2x\right)$

$2x-\frac{\pi}{3}=2k\pi+\frac{\pi}{2}-2x$ (1)

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$2x-\frac{\pi}{3}=2k\pi+\pi-\left(\frac{\pi}{2}-2x\right)$ (2)

, $k\in\mathbb{Z}$

(1) $\Leftrightarrow 2x+2x=2k\pi+\frac{\pi}{2}+\frac{\pi}{3} \Leftrightarrow$

$4x=2k\pi+\frac{5\pi}{6} \overset{:4}{\Leftrightarrow}$

$$x=\frac{k\pi}{2}+\frac{5\pi}{24}, k\in\mathbb{Z}$$

(2) $\Leftrightarrow$ ~~$2x$~~ $-\frac{\pi}{3}=2k\pi+\pi-\frac{\pi}{2}+$~~$2x$~~ $\Leftrightarrow$

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